DESIGN AND TEST OF A DYNAMIC BALANCING MACHINE FOR SMALL ROTORS

BY

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by
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PREFACE

Volumous amounts of work have been done on the design of sensitive dynamic balancing machines for rotors of various shapes and sizes. The degree of sensitivity obtained greatly complicates the design and adds to the cost of such a machine.

The purpose of this project is the design of a simple but yet satisfactory balancing machine which will detect and locate very small static and dynamic unbalances.

During the period from October, 1949 through April 1950, the author designed a novel type balancing machine and carried out tests to obtain experimental verification of the theoretical analysis used. Calibration curves were also plotted for future balancing problems in small rotors. This work was conducted at the United States Naval Post Graduate School.

The project was suggested by Professor Ernest K. Gatcombe and the author gratefully acknowledges his timely assistance and guidance throughout the entire project. Acknowledgements are also extended to the U. S. Naval Engineering Experiment Station for the manufacture of the machine, for the use of film and galvanometer units; to the David Taylor Model Basin for the use of Strain Indicators; to the Public Works Division of the U. S. Naval Academy for making tracings; to Mr. J. A. Oktavec of the Mechanical Force of the Post Graduate School for his workmanship in making special apparatus, and to other members of the faculty for loan of instruments.

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LIST OF SYMBOLS

W - Weight

Z - Weight

W_1 - Weight

Z_1 - Weight

C_L - Weight

C_R - Weight

a - Distance

L - Correction plane

R - Correction plane

M - Mass

μ - Mass per unit length

ℓ - Length

Y_o - Static deflection

X - Coordinate

Y - Coordinate

E - Modulus of elasticity

I - Moment of inertia

P.E - Potential energy

K.E - Kinetic energy

ω_n - Natural angular frequency

m - $\mu \ell$

h - Distance

K_1 - Spring constant of beam #1

K_2 - Spring constant of beam #2

θ - Angle of rotation

g - Acceleration of gravity

I_{cg} - Moment of inertia of the system about an axis in the xy plane through the center of gravity.

K_e - Equivalent spring constant for linear motion

K_{te} - Equivalent spring constant for torsional motion.

K_c - Coupling constant

f_1 - Natural frequency for linear vibration.

f_2 - Natural frequency for torsional vibration.

CHAPTER I

INTRODUCTION

It is impossible to manufacture a rotor in which the mass is so evenly distributed, that vibrations will not occur at the bearing supports. Therefore, to decrease bearing wear, failure due to vibration, etc., it becomes necessary to balance rotors both statically and dynamically.

A study of the engineering literature portrays many varieties of machines designed for static and dynamic balancing. They range from the comparatively simple machine of Laweczbeck Heyman, where the rotor is carried on spring supports, to the fine but complicated Gisholt Westinghouse machine which records the unbalance by a system of electrical networks.

In this day and age balancing requirements are so diverse that it is difficult to use one scheme for universal application. Therefore, it is not unusual to find that a new machine must be designed to accommodate a specific balancing problem.

The specific requirements for this machine is to accommodate light weight shafts ranging from two to four pounds in weight; seven to fifteen inches long and one quarter to one inch diameter.

Since the utmost requirement of a dynamic balancing machine is sensitivity, all effort throughout this project was funneled toward this requirement.

CHAPTER II

ELEMENTARY BALANCING CONSIDERATIONS

1. Nature of balancing.

Balancing operations are of two types, static and dynamic. Static balancing requires only the application of a single correction weight, whereas dynamic balancing requires two correction weights added in proper positions, in predetermined planes at some distance apart and perpendicular to the axis of rotation.

2. Static Unbalance.

Static unbalance sometimes called force unbalance, when of appreciable magnitude, is observed when the unbalanced part is mounted on horizontal knife edges as in Figure 1.

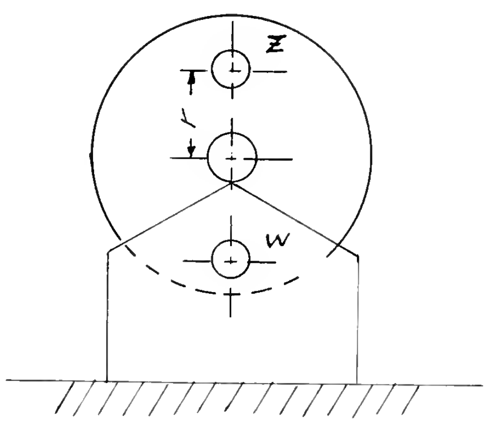


FIG 1

If the disc of Figure 1 has an unbalance equal to Wr oz-in ; a correction weight (Z) may be added 180° from W at such a distance from the center that $Zr = Wr$.

3. Dynamic Unbalance.

The presence of dynamic unbalance or moment unbalance occurs only when the piece is rotating. This type of unbalance is caused by two equal weights at equal distances from the rotational axis and located on opposite sides and opposite ends of the rotating shaft as in Figure 2.

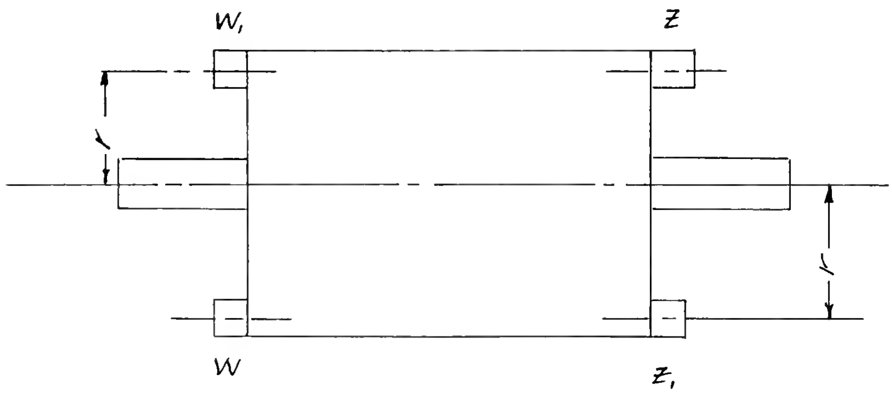


FIG 2

If in Figure 2, $Wr = Zr$ the piece will be in perfect static balance, but on rotation, centrifugal forces due to the weights will cause the ends of the cylinder to move in opposite directions. The piece will be in dynamic unbalance. By the addition of weights W_1 and Z_1 a compensating couple will be introduced and the piece will be in dynamic balance.

4. Static and Dynamic Unbalance.

Rotating parts usually have both static and dynamic unbalance. This is illustrated in Figure 3.

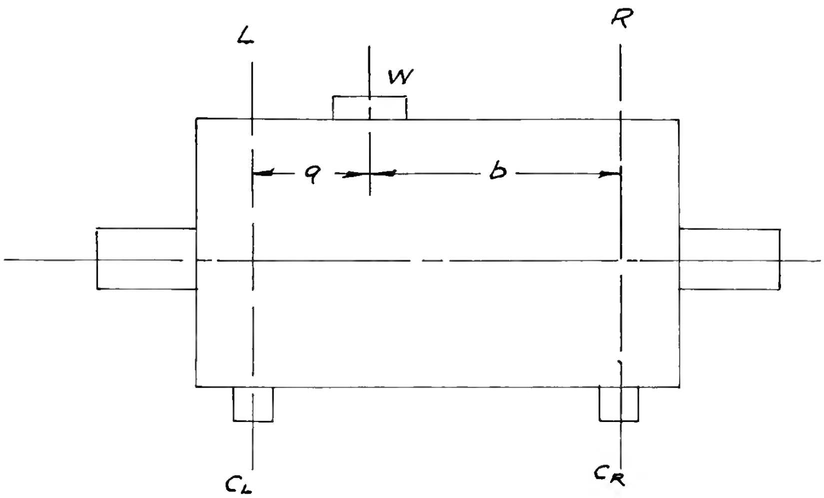


FIG 3

Let W in Figure 3 represent an unbalance in the rotor. The planes L and R are representative planes in which corrections can be added or removed.

If W is in the relative position as shown in Figure 3 two weights can be added in planes L and R to correct for both static and dynamic unbalance. For static balance $C_L + C_R = W$. For dynamic balance a moment equation of equilibrium can be satisfied.

CHAPTER III

MATHEMATICAL ANALYSIS

1. Problem.

The machine designed for this project can be simply described as two rectangular cantilever beams supporting a rotating shaft as illustrated in Figure 4.

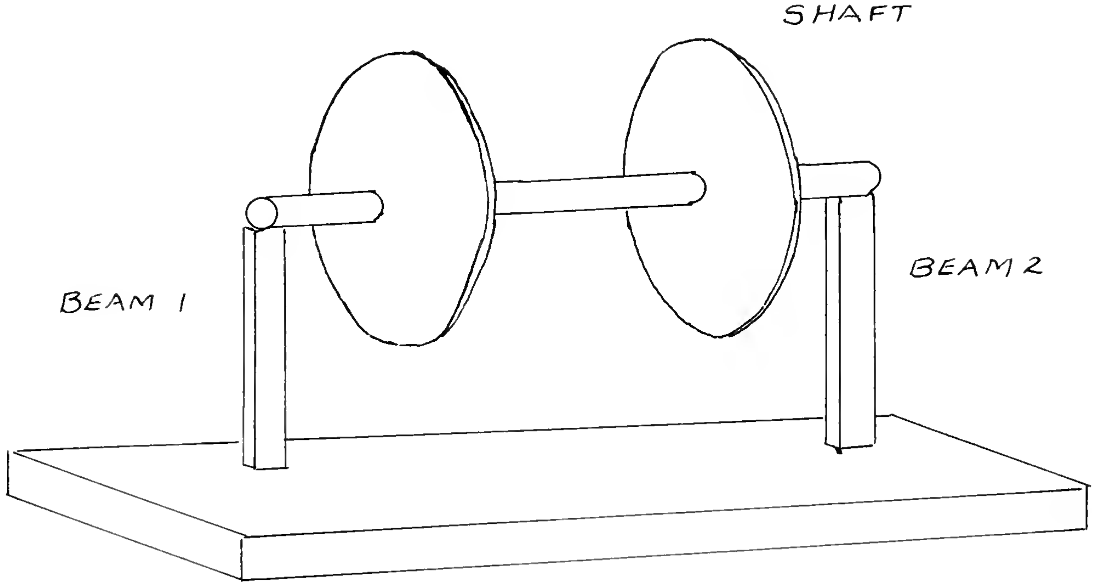


FIG 4

If the shaft in Figure 4 is both statically and dynamically unbalanced, the supporting beams 1 and 2 will experience a vibratory motion when the shaft is rotated. This vibratory motion will be extremely slight for minute unbalance, but if the system is forced to vibrate at it's natural frequency a maximum vibration, and therefore a max deflection of the beams

will occur.

However, to amplify the motion of the beams at resonant conditions of the system, requires that the natural frequency of the system be relatively high.

The mass of Beam#1, and Beam#2 must be considered when calculating the natural frequency of the system. To simplify matters, Rayleigh's method is used in finding what fraction of the beam's mass should be added to the concentrated mass supported at it's end.

The problem then is to determine the equivalent mass of each beam to be added to the concentrated mass they are supporting and to determine the natural frequency of the system.

2. Determination of the equivalent masses.*

In determining the equivalent masses, consider a cantilever beam EI of length (l) and mass (μ_1) per unit length (total mass $m = \mu_1 l$) carrying a concentrated load at its end as shown in Figure 5.

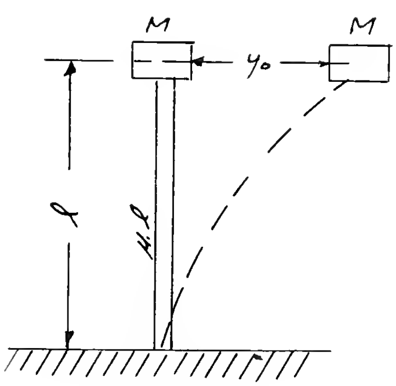


FIG 5

*See reference 3, 7.

Let mass (M) be deflected in the Y direction an amount y_o.
By Rayleigh's method (which is an approximation but sufficiently accurate for this problem), it is assumed that the deflection curve has the shape $y = y_o\left(1 - \cos\frac{\pi x}{\ell}\right)$ imposing the following end conditions:

$$x = 0, \quad y = 0$$

$$x = \ell \quad y = y_o$$

For the system the potential energy is

$$P.E. = \frac{EI}{2}\int_0^\ell \left(\frac{d^2y}{dx^2}\right)^2 dx \qquad \# \ (1)$$

The kinetic energy is

$$K.E. = \frac{1}{2} m_1 \omega_n^2 \int_0^\ell y^2 dx + \frac{1}{2} M \omega_n^2 y_o^2 \qquad \# \ (2)$$

By substituting the assumed deflection curve then,

$$P.E. = \frac{EI\, y_o \pi^4}{64\, \ell^3} \quad \text{and} \qquad (3)$$

$$K.E. = \frac{1}{2}\omega_n^2 y_o^2 \left[M + m\left(\frac{3}{2} - \frac{4}{\pi}\right)\right] \qquad (4)$$

By the theory of Conservation of Energy, the Kinetic Energy = Potential Energy, therefore equations (3) and (4) can be solved for a natural frequency.

$$\omega_n^2 = \frac{3.03\, EI}{\ell^3 (M + .23m)}$$

See appendix

Equation (5) shows that 23% of the mass $w_1 \ell$ of the beam should be added to the concentrated mass (M).

3. Equations of motion for the system*.

For setting up the equations of motion of the system a plan view of Figure 4 is utilized.

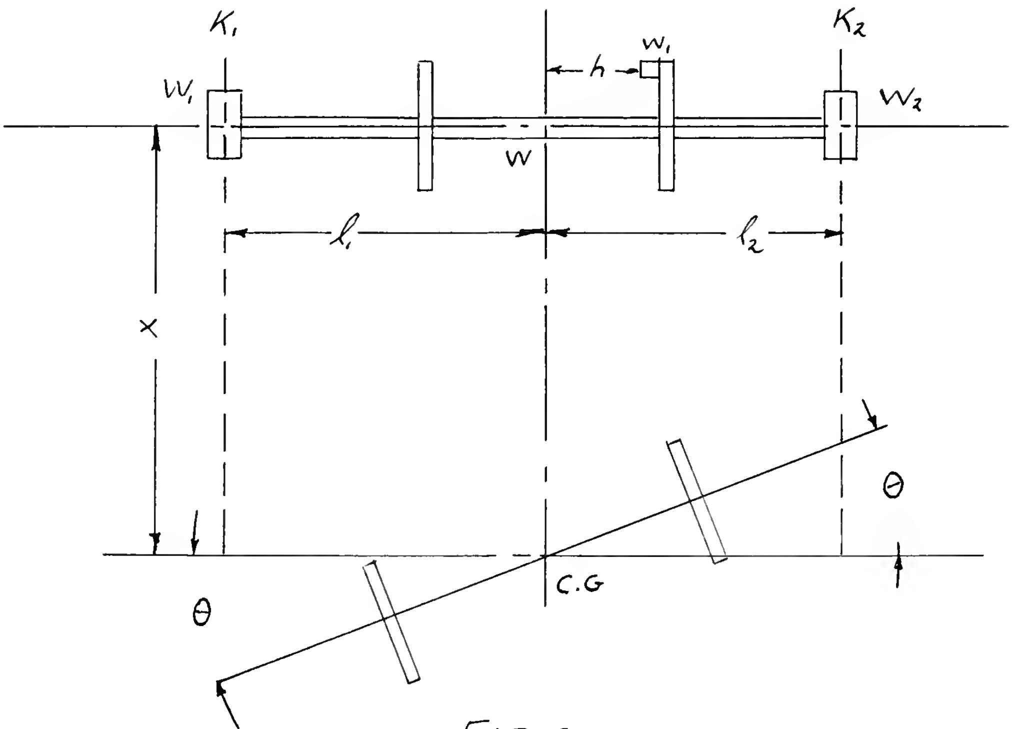


FIG 6

The system illustrated in Figure 6, depending upon the constraints placed on it can have two degrees of motion. If the system is displaced it can have linear motion in the (X) direction, and a rotation about the center of gravity if the condition exists such that $K_1 \ell_1 \neq K_2 \ell_2$.

Note* See reference 5.

In deriving the equations of motion the following assumptions are made:

1. The system is constrained to move in a linear direction along the (X) axis and a rotation in the xy plane about an axis through the center of gravity.
2. The rotor has sufficient stiffness that deflections in it are negligible.
3. Distances ℓ_1 and ℓ_2 from supports 1 and 2 remain constant.
4. The material is not stressed beyond its elastic limit and the beams do not deform in their built-in ends, thus their spring constants are linear.
5. Damping of the system is neglected.
6. Spring constants K_1 and K_2 are not equal.

When the system is displaced a distance x and rotated through an angle θ, beam 1 will deflect $x + \theta\ell_1$ and beam 2 will deflect $x - \theta\ell_2$. The forces exerted by beams 1 and 2 will respectively be $(x + \theta\ell_1)K_1$ and $(x - \theta\ell_1)K_2$.
Therefore the equation of motion in the x direction is

$$\frac{W}{g}\frac{d^2x}{dt^2} + x(K_1 + K_2) - (\ell_2 K_2 - \ell_1 K_1)\theta = 0 \tag{6}$$

By taking moments about the center of gravity the equation of motion in the torsional direction results in

$$I_{c.g}\frac{d^2\theta}{dt^2} + (K_2\ell_2^2 + K_1\ell_1^2)\theta - x(K_2\ell_2 - K_1\ell_1) = 0 \tag{7}$$

In equations (6) and (7) the following symbols are defined.

- W = total weight of the system
- g = acceleration of gravity
- x = linear displacement
- θ = angular displacement
- K_1 = Beam 1 spring constant
- K_2 = Beam 2 spring constant
- l_1 = distance of Beam 1 from c.g.
- l_2 = distance of Beam 2 from c.g.
- $I_{c.g}$ = moment of inertia of the system about an axis in the xy plane through the center of gravity

~~c.g.~~ c.g. = center of gravity of the system

4. Solution for the natural frequency of the system.

A solution of equation (6) and (7) can be obtained in the form

$$x = x_0 \sin \omega t$$

$$\theta = \theta_0 \sin \omega t$$

Making these substitutions result in:

$$-\frac{W}{g} x_0 \omega^2 + x_0 (K_1 + K_2) - \theta_0 (l_2 K_2 - l_1 K_1) = 0 \qquad (8)$$

and

$$-I_{c.g}\, \theta_0 \omega^2 + (K_2 l_2^2 + K_1 l_1^2)\theta_0 - (K_2 l_2 - K_1 l_1) x_0 = 0 \qquad (9)$$

It is seen that K_1+K_2 will be the equivalent spring constant for linear motion; for simplification let $K_e = K_1+K_2$. The

equivalent spring constant for torsional motion is

$$K_{te} = (K_2 \ell_2^2 + K_1 \ell_1^2)$$

If $(K_2 \ell_2 - K_1 \ell_1) = K_c$ where K_c is designated as the coupling constant, then equations (8) and (9) reduce to

$$\frac{W}{g} X_0 \omega^2 - K_e X_0 + K_c \theta_0 = 0 \qquad (10)$$

$$I_{c.g} \theta_0 \omega^2 - K_{te} \theta_0 + K_c X_0 = 0 \qquad (11)$$

Eliminating X_0 and θ_0 results in the equation for the natural frequencies of the system.

$$\omega^4 - \left(\frac{K_e g}{W} + \frac{K_{te}}{I_{c.g.}}\right)\omega^2 + \left(\frac{K_e K_{te} - K_c^2}{W I_{c.g.}}\right) g = 0 \qquad (12)$$

When $K_2 \ell_2 = K_1 \ell_1$ the coupling constant is zero and equations (10) and (11) reduce to their uncoupled form. When the vibrations are not coupled each acts independently of the other. For this special case the following frequencies would result.

$$f_1 = \frac{1}{2\pi} \sqrt{\frac{K_e g}{W}} \qquad (13)$$

$$f_2 = \frac{1}{2\pi} \sqrt{\frac{K_{te}}{I_{c.g.}}} \qquad (14)$$

In general the coupling will raise the higher frequency and lower the lowest frequency. If the coupling is small the effect on frequency is not very great.

5. Amplitude of vibration.

If in Figure 5 a small unbalanced weight (w_1) exists in the rotor and is located a distance (h) from the center of gravity, the system can be forced to vibrate at a natural frequency.

The equations for forced vibration neglecting damping are:

$$\frac{W}{g}\frac{d^2x}{dt^2} + x(K_1+K_2) - (\ell_2K_2 - \ell_1K_1)\theta = \frac{w_1}{g} r\omega^2 \sin\omega t \qquad (15)$$

$$I_{c.g.}\frac{d^2\theta}{dt^2} + \theta(K_2\ell_2^2 + K_1\ell_1^2) - x(K_2\ell_2 - K_1\ell_1) = \frac{w_1}{g} r\omega^2 h \sin\omega t \qquad (16)$$

A solution can be in the form

$$x = A \sin\omega t$$

$$\theta = C \sin\omega t$$

Substituting the symbols for the spring constants, the constants A and C are determined as

$$A = \frac{\frac{w_1}{g} r\omega^2 \left[\left(K_{te} - I_{c.g}\omega^2 + hK_c\right)\right]}{\left[\left(K_e - \frac{W}{g}\omega^2\right)\left(K_{te} - I_{c.g}\omega^2\right) - K_c^2\right]}$$

$$C = \frac{\frac{w_1}{g} r\omega^2 \left[K_c + h\left(K_e - \frac{W}{g}\omega^2\right)\right]}{\left[\left(K_{te} - I_{c.g}\omega^2\right)\left(K_e - \frac{W}{g}\omega^2\right) - K_c^2\right]}$$

Therefore

$$x = \frac{\frac{w_1}{g} r\omega^2 \left[K_{te} - I_{c.g}\omega^2 + hK_c\right]}{\left[\left(K_e - \frac{W}{g}\omega^2\right)\left(K_{te} - I_{c.g}\omega^2\right) - K_c^2\right]} \sin\omega t. \qquad (17)$$

$$\theta = \frac{\frac{W_1}{g} r\omega^2 \left[K_c + h\left(K_e - \frac{W}{g}\omega^2\right)\right]}{\left[\left(K_{te} - I_{c.g}\omega^2\right)\left(K_e - \frac{W}{g}\omega^2\right) - K_c^2\right]} \sin\omega t \tag{18}$$

Since Beam 1 deflects $x + \theta\ell_1$ then

$$x_1 = \frac{\frac{W_1}{g} r\omega^2 \sin\omega t \left[K_{te} - I_{c.g}\omega^2 + hK_c\right] + \ell_1\left[K_e + h\left(K_e - \frac{W}{g}\omega^2\right)\right]}{\left[\left(K_{te} - I_{c.g}\omega^2\right)\left(K_e - \frac{W}{g}\omega^2\right) - K_c^2\right]} \tag{19}$$

Beam 2 deflects $x - \theta\ell_2$, then

$$x_2 = \frac{\frac{W_1}{g} r\omega^2 \sin\omega t \left[K_{te} - I_{c.g}\omega^2 + hK_c\right] - \ell_2\left[K_c + h\left(K_e - \frac{W}{g}\omega^2\right)\right]}{\left[\left(K_{te} - I_{c.g}\omega^2\right)\left(K_e - \frac{W}{g}\omega^2\right) - K_c^2\right]} \tag{20}$$

The deflections may be expressed as ratios:

$$\frac{x_2}{x_1} = \frac{\left[K_{te} - I_{c.g}\omega^2 + hK_c\right] - \ell_2\left[K_c + h\left(K_e - \frac{W}{g}\omega^2\right)\right]}{\left[K_{te} - I_{c.g}\omega^2 + hK_c\right] + \ell_1\left[K_c + h\left(K_e - \frac{W}{g}\omega^2\right)\right]} \tag{21}$$

CHAPTER IV

APPROACH TO THE DESIGN PROBLEM

In designing a balancer capable of detecting and locating very small unbalances, certain important factors and problems which do not exist in more common balancing operations must be considered.

1. Problems in Design.

The problems encountered are:

(1) Designing into the machine a very high degree of sensitivity.

(2) Spinning the rotor in the balancing fixture. Mechanical couplings or belts introduce an unbalanced moment in the shaft after it has been removed from the balancer. Electrical drive is some times impossible or impracticable.

(3) Proper selection of bearings, such that they absorb the minimum of energy.

(4) Selecting a material for vibrating members which will withstand cyclic stresses without failure and have inherently low internal damping capacity.

2. Sensitivity of the machine.

Sensitivity may be defined as the amplitude of vibration of the system per unit unbalanced mass at unit radius.

Probably the most important factor affecting sensitivity is the overall damping. This presents a most difficult

situation and can hardly be solved without recourse to preliminary investigation. This investigation was beyond the scope of the project.

The overall damping is due to a variety of causes such as (a) air resistance (b) damping of supports (c) damping due to bearings (d) internal damping in the metal of the elastic members (e) energy loss in the foundation. Maximum sensitivity depends on reducing these to a minimum.

Damping (a) can usually be considered small, nevertheless, it still is impracticable to remove. In the case of (b) supports must be firmly fixed and fits between mating surfaces must be extremely close. In the case of (c) ball bearings should be avoided if possible, or very high precision instrument~~s~~ bearings with small clearances and minimum friction should be used. Perhaps the most important aspects are those of (d) and (e).

The main object in designing the suspension system i~~f~~s the choice of a material of low internal damping capacity. Although there are volumes of material published concerning the damping capacity of metals, knowledge in this sphere is still limited. Following is an exerpt taken from a letter received by the author from the Aluminum Company of America.

"From work that has been done on the subject it is generally accepted that aluminum alloys have extremely low damping capacities. Also it is definitely indicated that the damping capacity of most structures or assemblies is much more a function of the joints and connections than of the internal damping capacity of the material."

The main value of the published data is in showing that certain materials, mainly brasses, bronzes, duraluminum and hardened steel have low damping capacities, while ductile materials, cast metals and plastics exhibit higher damping characteristics..

The base upon which the suspension system is attached should be of large mass and high stiffness so that its natural frequency of vibration is well above that of the suspension.

CHAPTER V

APPARATUS

Detailed plans of the balancing machine designed by the author are enclosed, attached to the back cover in Folder #1.

1. Suspension System.

A relatively high natural frequency of the system dictated the use of a light material for the vibrating members. The suspension system consists essentially of two cantilever beams extending vertically from the base. The beams, with a cross-section of 1/2" x 3/4", were constructed from duraluminum (24ST-4). They extend into the base with very close snug fits and anchored with nuts.

It was necessary to machine the beams to very close tolerances so their spring constants would be similar. To have them exactly equal, constitutes a difficult problem. The spring constant is affected not only by the dimensions but also by the non-honogeniety of the heat treatment, and differences in the fits of mating parts.

2. Power System.

Since mechanical couplings or belts used for spinning the rotor in the balancing fixture introduce unbalanced moments, small air driven wheels were used. Each end of the rotor journal rests between two such wheels, thus eliminating any unbalanced moment. The wheels were not machined integrally

due to problems arising in milling out the small buckets which are spaced at 7° intervals around the periphery. The buckets were milled in a solid aluminum disk and two flanges were riveted to the disk for complete assembly. The method is illustrated in Figure 7.

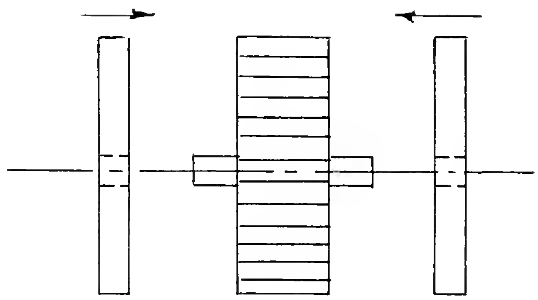


FIG 7

- The driving wheels are supported on R-2 Inch Series New Departure ball bearings (1/8" I.D; 3/8" O.D.) They are driven simultaneously by compressed air which follows a path through a 1/8" drilled hole in the cantilever beams and cross supports. It is then directed through 3/32" drilled holes in the wheel support brackets leading to 3/32" diameter nozzles which project air into the buckets at a 30° angle with the vertical.

3. Base.

The base of the machine was designed exceedingly heavy to eliminate the necessity of bolting it down. A heavy base is also advantageous in that its natural frequency is well above

that of the suspension system. Drilled and reamed holes were provided to accommodate rotors of various length.

4. Test Shaft.

A test shaft was designed for calibrating the machine in both static and dynamic unbalance. Two disks which provide correction planes, in which known weights can be added, where welded on the journal so they would be in close proximity to the supports to produce maximum vibrations. Holes 1/8" diameter where drilled and tapped in the face of the disks; spaced every 10° to accommodate small aluminum correction weights.

5. Protection device.

Since the rotor is free to move vertically upward off it's driving wheels if the centrifugal force due to unbalances is sufficient, it was necessary to provide for protection. The protecting guard consists of two flat steel bars extending across the rotor and supported at the base. The height of the guard can be adjusted to accommodate different size rotors.

6. Vibration Pickup System.

From the theory of Strength of Materials a cantilever beam with a concentrated mass on its end will have its maximum flexural stress and therefore maximum strain at the built-in end. Using this fact as a basis a system was set up to measure the strains produced in the beams by the unbalanced rotor.

A relatively new, but very proficient system is the use of wire strain gages. Figure 8 illustrates the type of gage used.

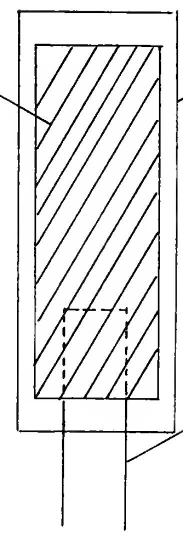


FIG 8

The gage is of the A-1 SR-4 type manufactured by Baldwin Locomotive Works, Philadelphia, Pa.

The fine wire forming the strain sensitive element is usually constructed of .001" dia wire of some material having a low thermal coefficient of resistivity. Alloys of the constantin type are frequently used. The sensitive wire is covered with a square piece of felt and supported by thin paper having good mechanical and insulating characteristics.

The gage operates on the principle that a change in length of sensitive wire produces a proportional change in its electrical resistance. This fact may be expressed by

$$\frac{\Delta R}{R} = K \frac{\Delta l}{l}$$

Where R is the electrical resistance of the gage

Δr is the change in R due to strain

l is the length of gage

Δl is the change of length

K is a constant of proportionality.

To make an accurate measurement of the strain in the beams a carrier type strain indicator was used.

The TMB type IA strain indicator developed by the David Taylor Model Basin, utilizes two A-1 SR-4 strain gages with a gage resistance of 120 ohms and a gage factor K of 2. They are connected as adjacent arms in an A.C. bridge circuit; the block diagram of the system is illustrated in Figure 9.

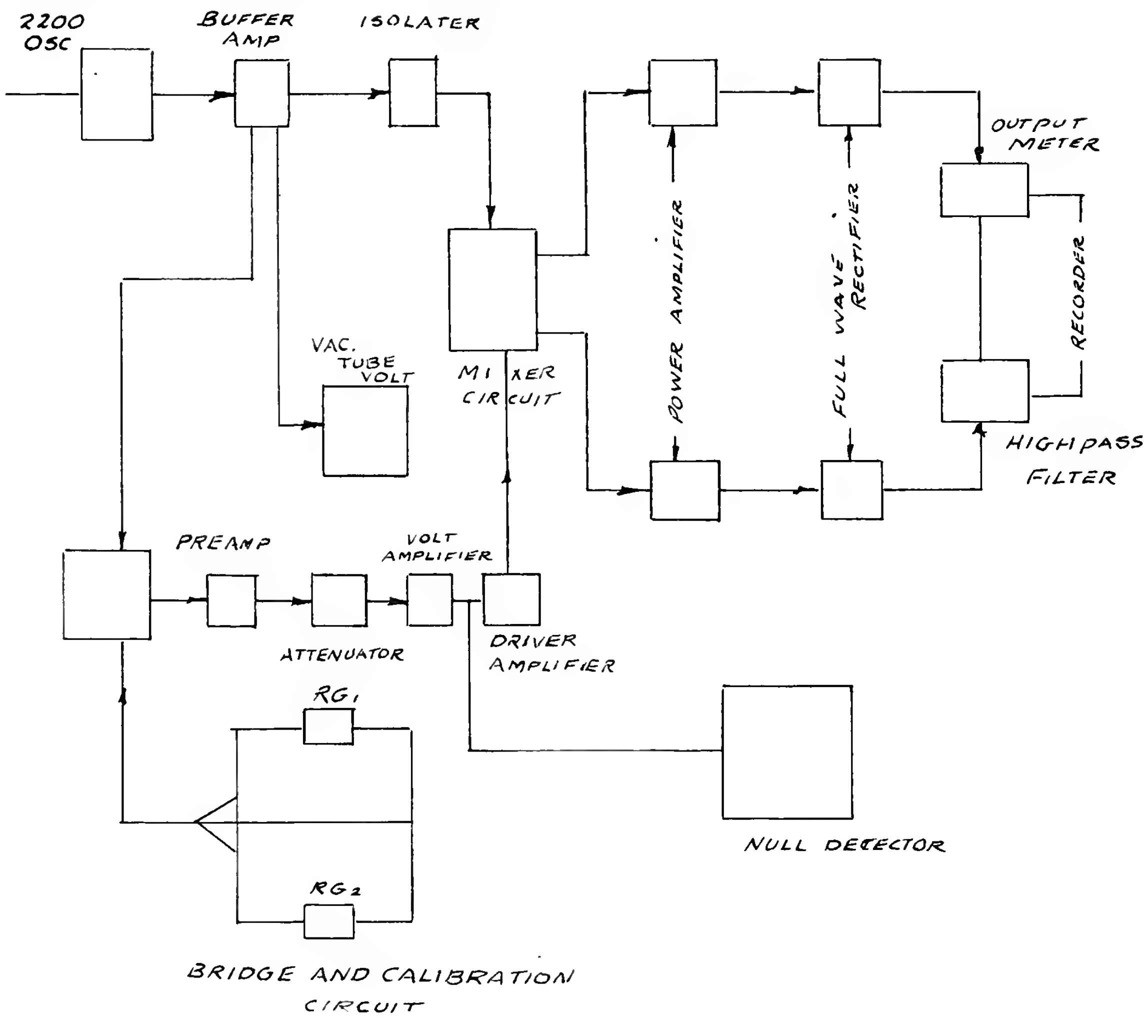


FIG 9

The indicator can be calibrated to read from 40 micro inches/inch to 2500 micro inches per inch strain with an accuracy of $\pm$ .25%

For detail discussion on design and operation see reference #2 in Bibliography.

7. Recorder.

The output of the strain indicator was fed into a Universal Type P.A. Westinghouse Oscillograph. Since the max output of the indicator is 4 mill. amps, a super-sensitive string galvanometer sensitive to 23 mill. amps was used. Only one such element could be located in the area, therefore simultaneous readings for each beam was impossible.

When two gages are used on each beam, the recorder indicates double strain.

8. Arrangement of Apparatus.

Figure 10 shows an overall view of apparatus. Figure 11 gives a close up view of balancing machine. The recording machine is shown in Figure 12.

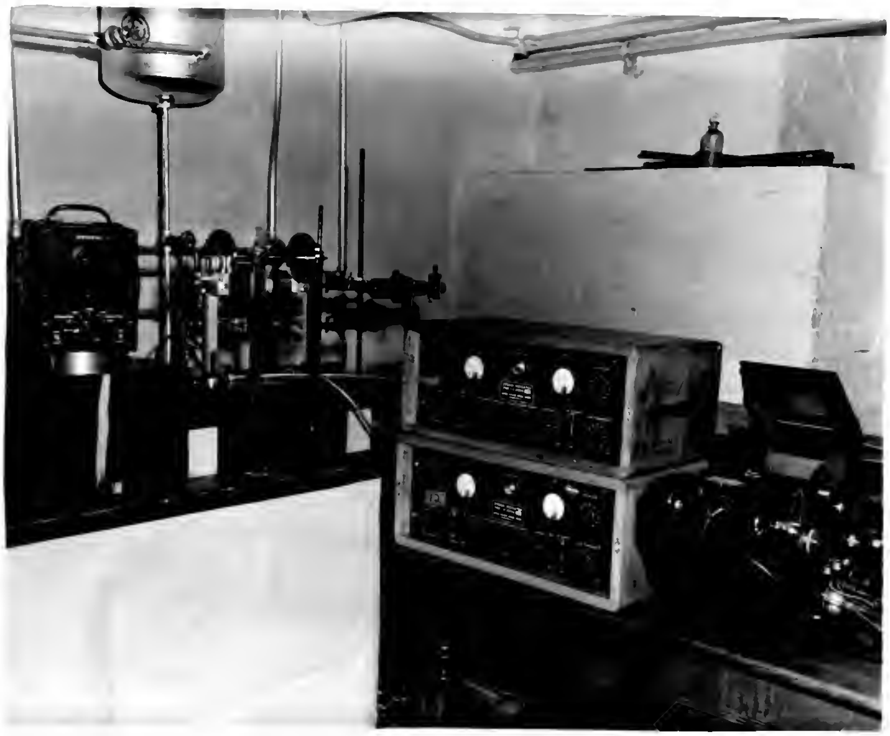


FIG 10

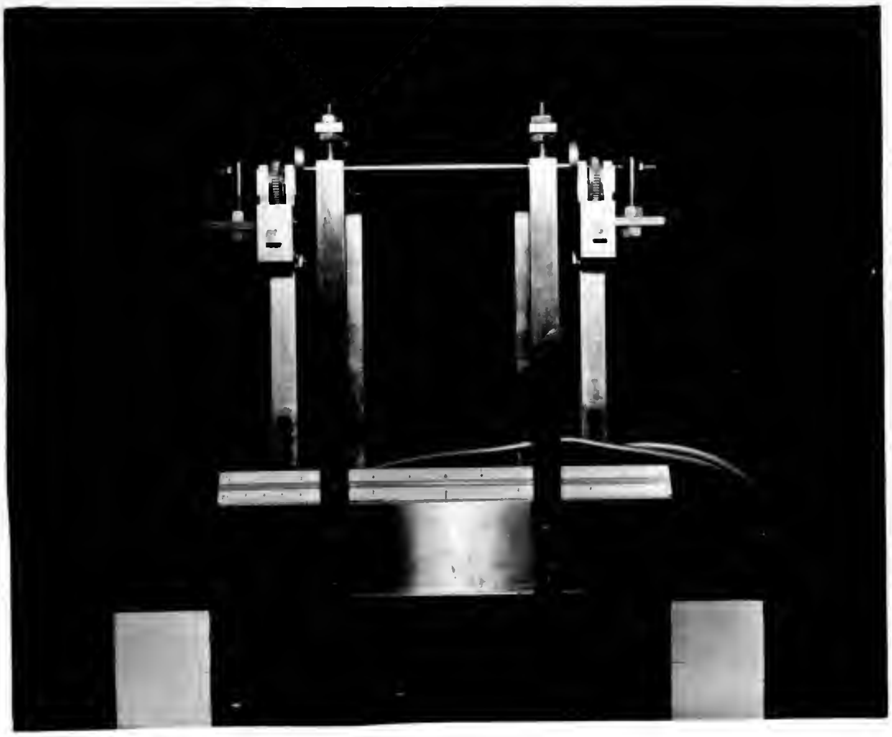


FIG 11.

FIG 12

CHAPTER VI

PROCEDURE AND RESULTS

1. Preliminary tests.

Before running any tests on the machine, it was necessary to determine the mass of each part and their centers of gravity. From this data the moments of inertia were calculated. The spring constants of the beams were determined experimentally. The test shaft, not in perfect balance when manufactured, was dynamically balanced as best could be, considering that time was very limited.

2. Masses and centers of gravity.

Each component of the machine was weighed to the nearest 1/100 of a gram. The weights are listed in Table I, opposite to a sketch of each part.

Due to the complicity of the design, it was necessary to determine the centers of gravity by an experimental method. This consisted of finding the balance point of the parts or assembly of parts, by placing them on a knife edge which rested on a level plane. The centers were measured to the nearest 1/100".

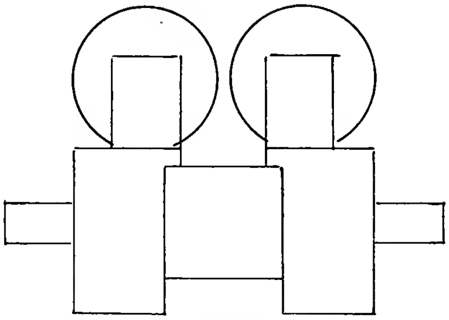
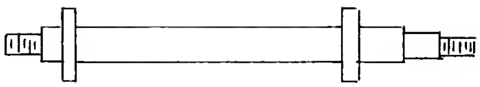
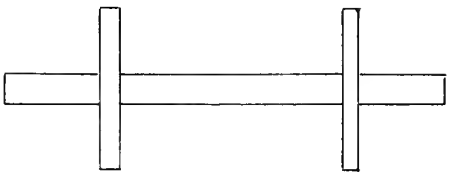
PART SKETCH	WEIGHT- GRAMS
 CROSS SUPPORT AND WHEEL ASSEMBLY 1.	296.90
CROSS SUPPORT AND WHEEL ASSEMBlY 2.	296.70
 BEAM 1	173.43
 TEST ROTOR	1252.10
BEAM 2	173.35

TABLE I

3. Spring constants.

An attempt was made to measure the static deflection of the beams by an optical telescope. This proved unsuccessful since the deflections could be readonly to the nearest .0005".

An Ames-Jeweled Bearing Dial Indicator Gage which recorded to the nearest 1/10,000 inch deflection did however prove successful. Before making the measurements the machine was assembled and aligned, then turned on its edge and clamped to a drill press table. The beams were loaded statically such that the line of force of the load passed through the center line of the shaft. Tables II and III summarize data recorded for each beam. Figures ~~14~~ 13 and ~~15~~ 14 show the graphs of deflection plotted against load.

The spring constants determined:

$$K_1 = 335 \text{ lb/in}$$

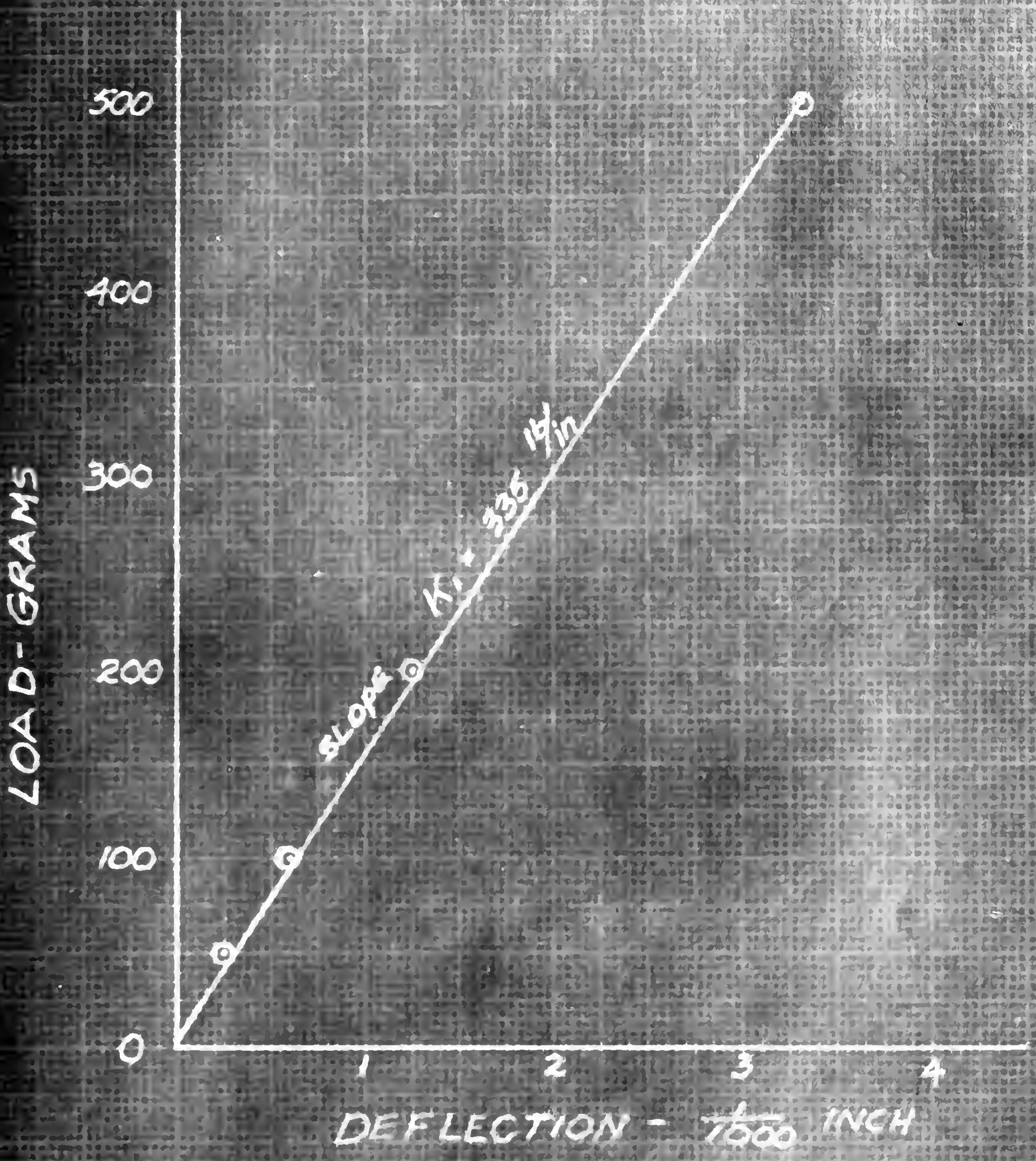
$$K_2 = 316 \text{ lb/in}$$

Load (grams)	Deflection (inches)
50	.00025
100	.00060
200	.00125
500	.00325

TABLE II
Data for Beam 1

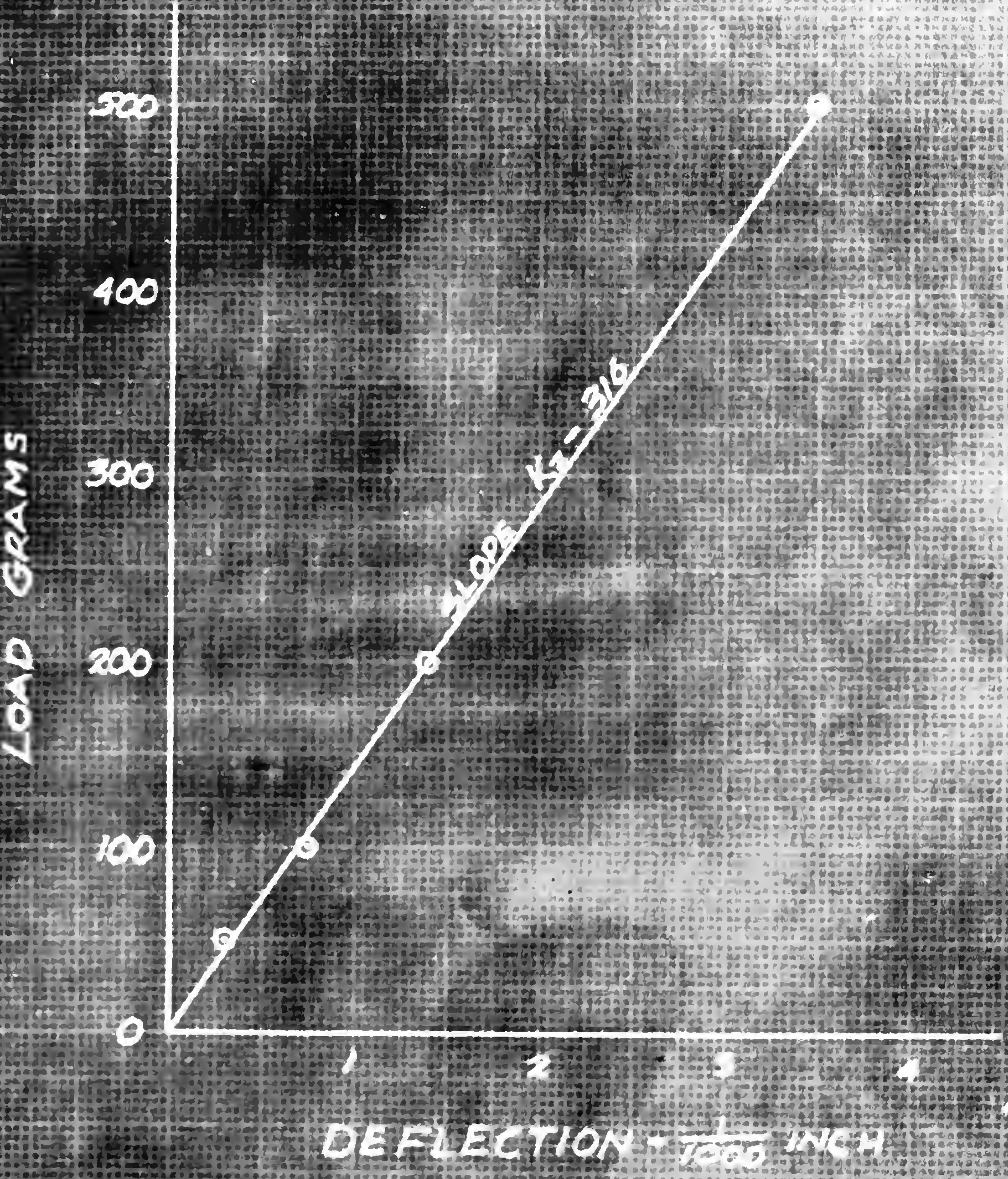
Load (grams)	Deflection (inches)
50	.00030
100	.00075
200	.00140
500	.00350

TABLE III
Data for Beam 2



SPRING CONSTANT CURVE
FOR
BEAM 1

FIG 13



SPRING CONSTANT CURVE
FOR
BEAM 2

FIG 14

4. Natural Frequency.

The natural frequencies of the system were determined experimentally by spinning the rotor to a speed above the resonant condition and then letting it decelerate slowly. When the amplitude of vibration reached a maximum as recorded on the oscillograph, the speed was noted on the strobotac. The system, as the theory predicted, experienced two natural frequencies. The lowest at a speed of 2410 R.P.M. was due to linear vibration and the highest at speed of 3000 R.P.M. was due to torsional vibration.

Theoretically, the frequencies were calculated at $f_1 = 43$ cps or 2580 rpm and $f_2 = 51.7$ cps or 3100 rpm.

These frequencies can be calculated in the following manner using the theory in Chapter II.

The total weight of vibrating beam is .2456 lbs. and its center of gravity is 3.4 inches from the base as shown in Figure 15.

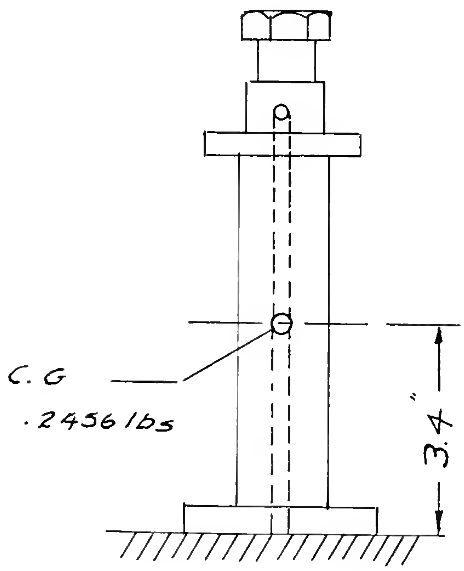


FIG 15

The total weight supported by each beam, disregarding the weight of the shaft, is .870 lbs. This includes weight of wheels, supports cross support and end bearings and holders. The center of gravity of the beam assembly including the concentrated mass is 6.74" from the base as shown in Figure 16.

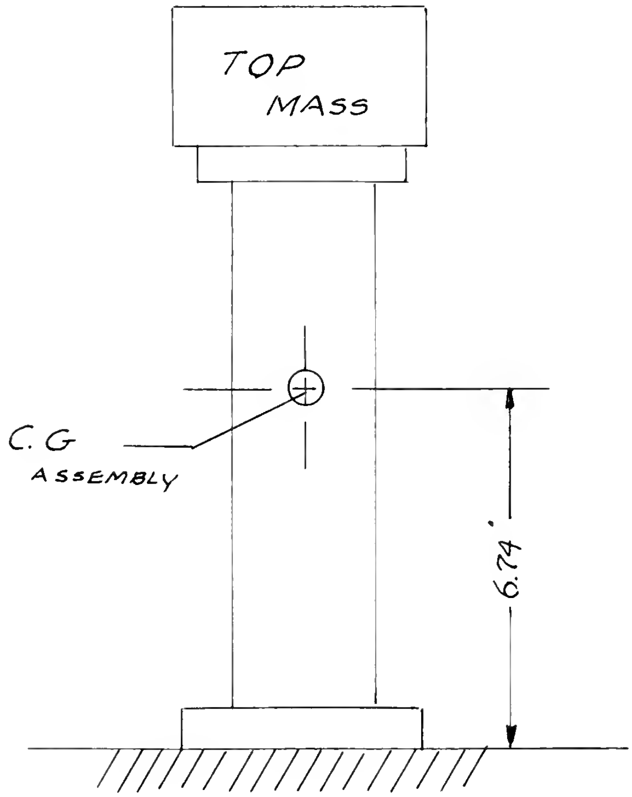


FIG 16

According to Timoshenko and DenHartog, as shown in equation 5, Chapter II, the system can be considered as a cantilever beam supporting a total load consisting of the concentrated load

plus 23% of the weight of the beam.

These considerations reduced the original system to an equivalent consisting of a mass less beam of length 6.74" supporting a load of .926 lbs. This is represented in Figure 17.

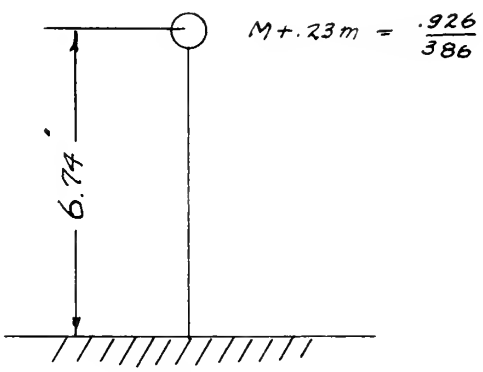


FIG 17

If the above system is displaced its natural frequency is:

$$\omega_{n_1}^2 = \frac{3.03\,EI}{\ell_1^3 (M + .23m)} \qquad (\text{equation 5 Chapt. II})$$

where E and I are considered constant over length (ℓ).

Now, consider the weight of the shaft supported by the beams. This weight is evenly distributed between the beams and is concentrated 9.375" above the base.

Therefore, the mass (M + .23m) must be transferred to this point such that the resulting system will vibrate at the same frequency as that shown in Figure 17. This transfer can be illustrated as shown in Figure 18.

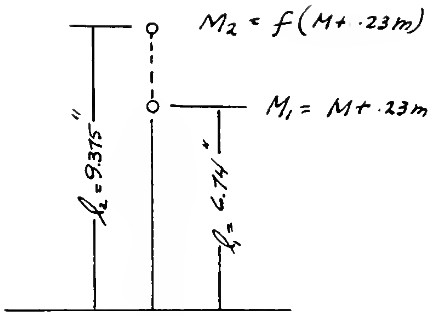


FIG 18

By using the same equation as before, then

$$\omega_{n_2}^2 = \frac{3.03\,EI}{\ell_2^3\,(M_2)}$$

Since $\omega_{n_2}^2 = \omega_{n_1}^2$ then

$$\frac{3.03\,EI}{\ell_2^3\,M_2} = \frac{3.03\,EI}{\ell_1^3\,M_1} \quad \text{or} \quad \frac{M_2}{M_1} = \frac{\ell_1^3}{\ell_2^3} \;; \quad M_2 = \frac{\ell_1^3}{\ell_2^3}\,M_1$$

Substituting the appropriate values.

$$M_2 = \left(\frac{6.74}{9.375}\right)^3 (.926) = .344 \text{ lbs.}$$

Therefore .344 lbs. must be added to the weight of the shaft or the total weight supported by each beam = .344+1.385 = 1.729 lbs.

Before calculating the natural frequencies of the system by equation 12, Chapter II, the moment of inertia of the system about its center of gravity must be determined. Let Figure 19 represent the shaft and the point concentrated masses supported by the beams.

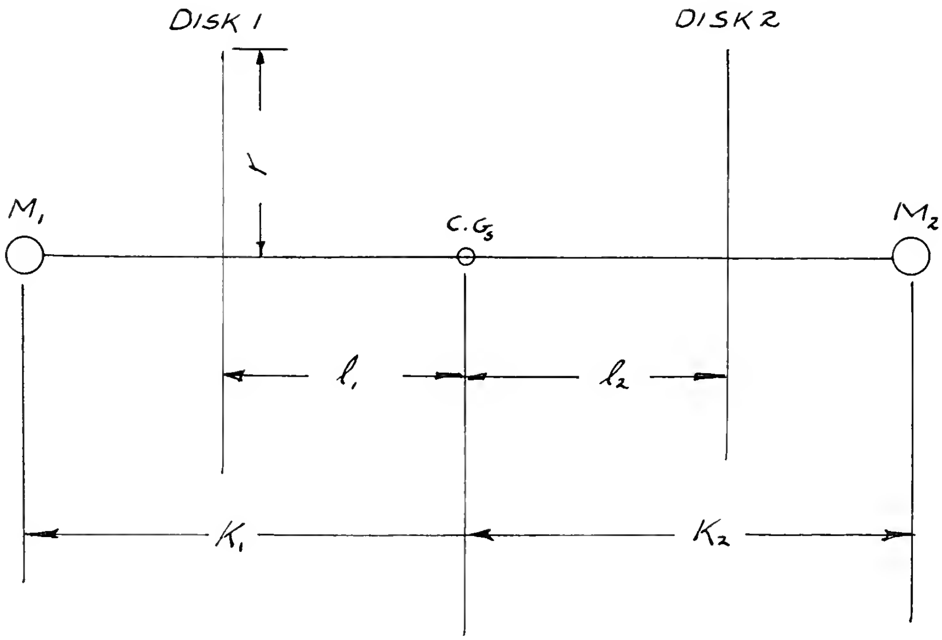


FIG 19

For disks $I_{c.g_s} = 2\left[M_{disk}\dfrac{r^2}{4} + M_{disk}\,C_1^2\right] = .070$ units

For $M_1 + M_2$ $\quad I_{M_1+M_2} = 2\left(M_1 \ell_1^2\right) = .0304$ units

Total $I_{c.g_s} = .104$ units

Total weights of system = 3.458 lbs.

Using the following equation to find ω_n

$$\omega_n^4 - \left(\frac{K_e g}{W_T} + \frac{K_{te}}{I_{c.g.}}\right)\omega_n^2 + g\,\frac{\left(K_e K_{te} - K_c^2\right)}{W_T I_{c.g}} = 0$$

Substituting the following values:

$K_e = 316 + 335 = 651$

$K_c = (335 - 316)(4.126) = 785$

$K_{te} = (651)(4.126)^2 = 11{,}100$

$W_t = 3.458$

$I_{c.g} = .104$

$g = 386$

The resulting frequencies are f_1 (linear) = 2580 c.p.m.

f_2 (torsional) = 3100 c.p.m.

Summary.

Calculated values	Experimental values.
f_1 = 2580 or 43 cps	f_1 = 2410 or 40 cps
f_2 = 3100 or 51.7 cps	f_2 = 3020 or 50.3 cps

5. Calibration.

Before running calibration tests the rotor was balanced as nearly perfect as possible. This was a trial and error method of adding very small weights of putty in both disks.

To illustrate the method of calibration refer to Figure 20.

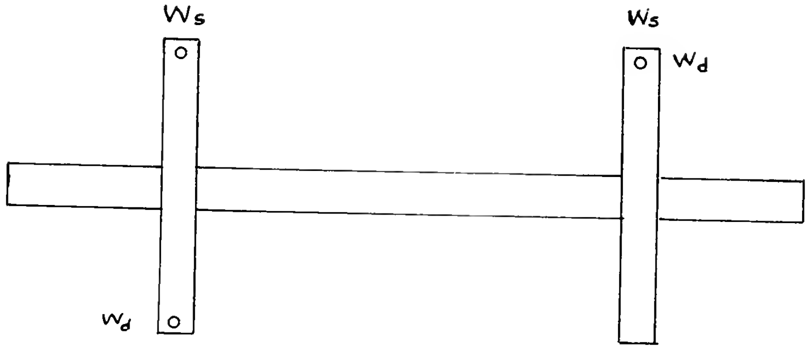
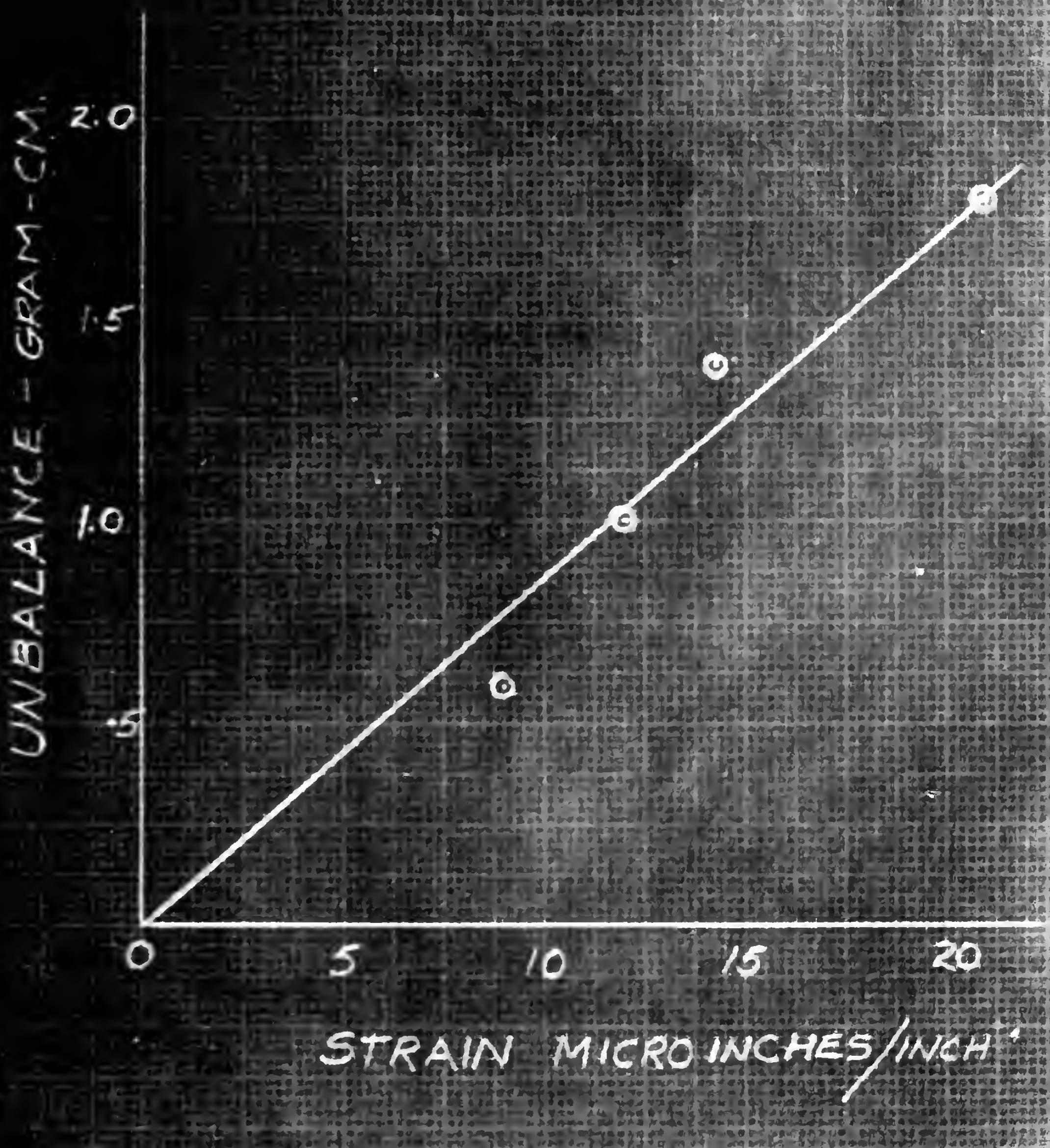


FIG 20

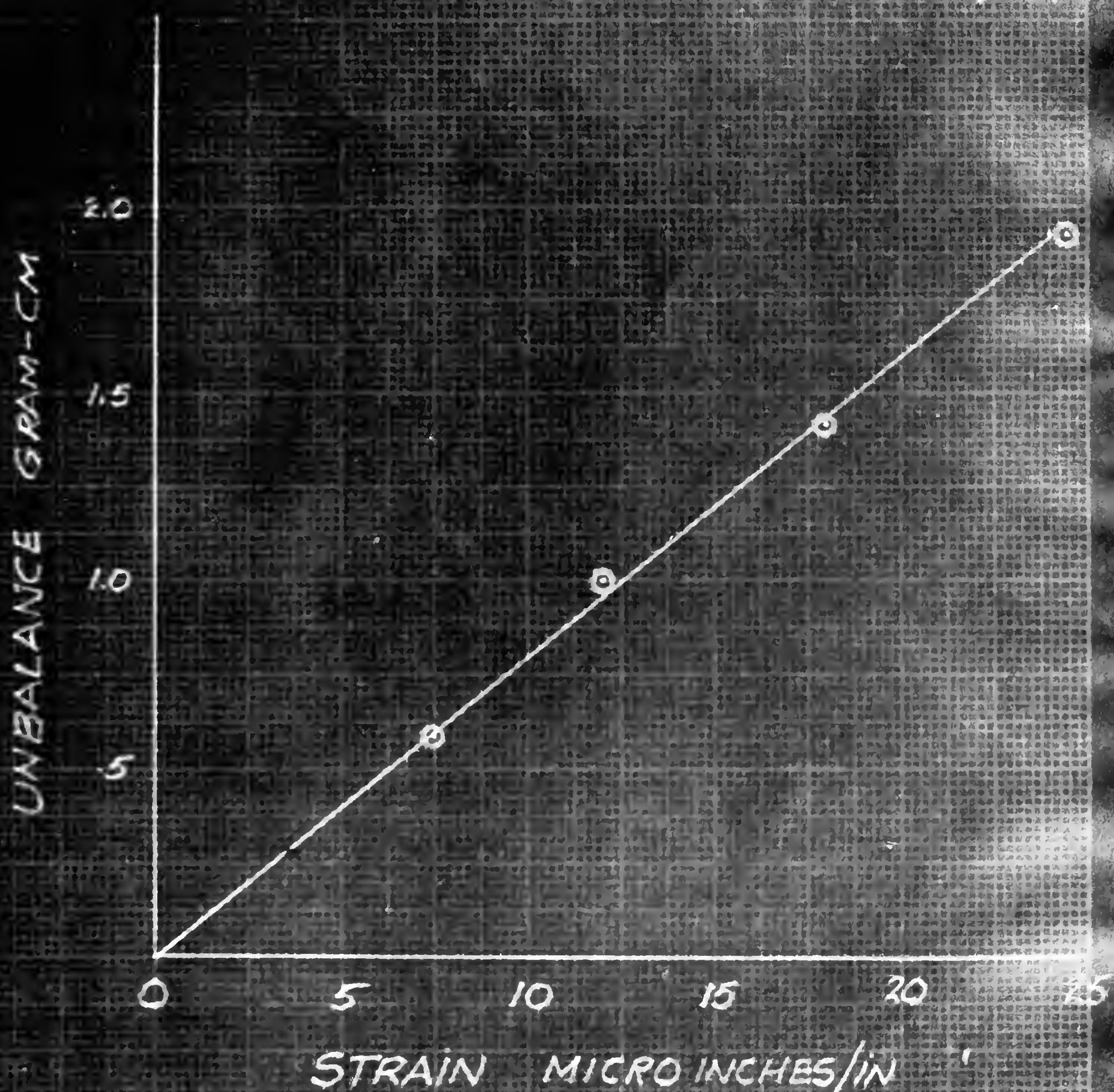
Static calibration was conducted by adding equal weights W_s as shown in Figure 20 at the same angle in each disk. The rotor was decelerated very slowly through the natural frequency. The high amplitude of vibration which exists near resonance is the result of the energy fed into the system by the unbalanced force, and obviously depends on the time taken in passing through the critical range. Therefore, critical speed control is necessary, Photographs were taken when the amplitude was maximum. The oscillographs for each beam are shown on the following pages. To obtain the data for the calibration curves, the zero unbalance condition was used as a base. The amount of strain produced by each unbalance weight was determined by taking the differences in amplitudes. Since double strain was recorded the results were halved. Results are shown in Table IV and V. Figures 21 and 22 are the calibration curves.



STATIC CALIBRATION

BEAM I

FIG 21



STATIC CALIBRATION
BEAM 2

FIG 22

Unbalance (gram-cm)	Strain (micro inches/inch)
0	0
.594	9.00
1.005	12.00
1.395	14.25
1.800	21.00

TABLE IV
Beam #1 - Static

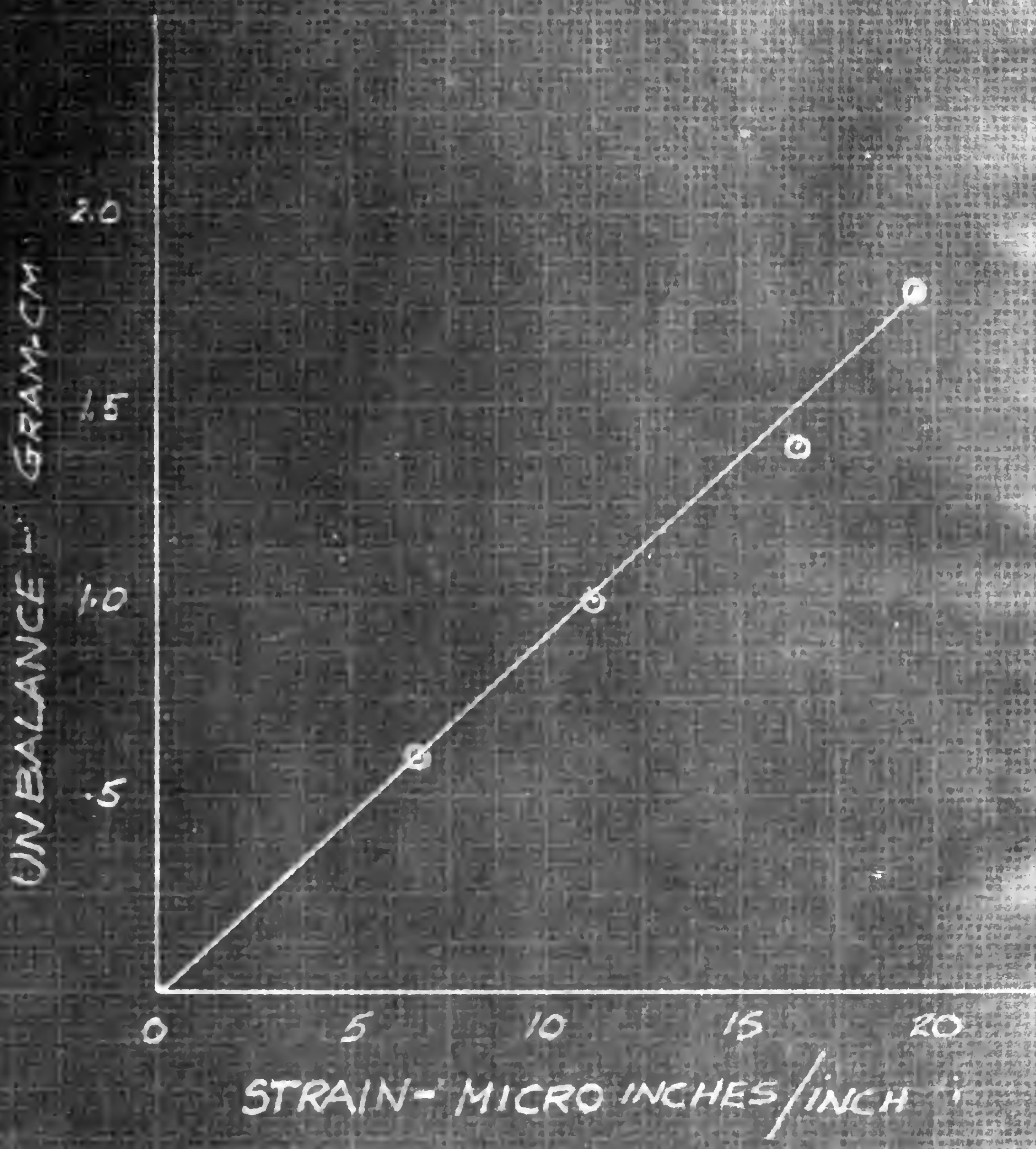
Unbalance (gram-cm)	Strain (micro inches/inches)
0	0
.594	7.5
1.005	12.0
1.395	18.0
1.800	24.5

TABLE V
Beam #2 - Static

Dynamic calibration was conducted in the same manner as described for static, except two equal weights were added, one in each disk 180° out of phase as shown in Figure 20. Results are shown in Tables VI and VII. Figures 23 and 24 are the calibration curves.

Unbalance (gram-cm)	Strain (micro inches/inch
0	0
.594	6.75
1.005	11.25
1.395	16.50
1.800	19.50

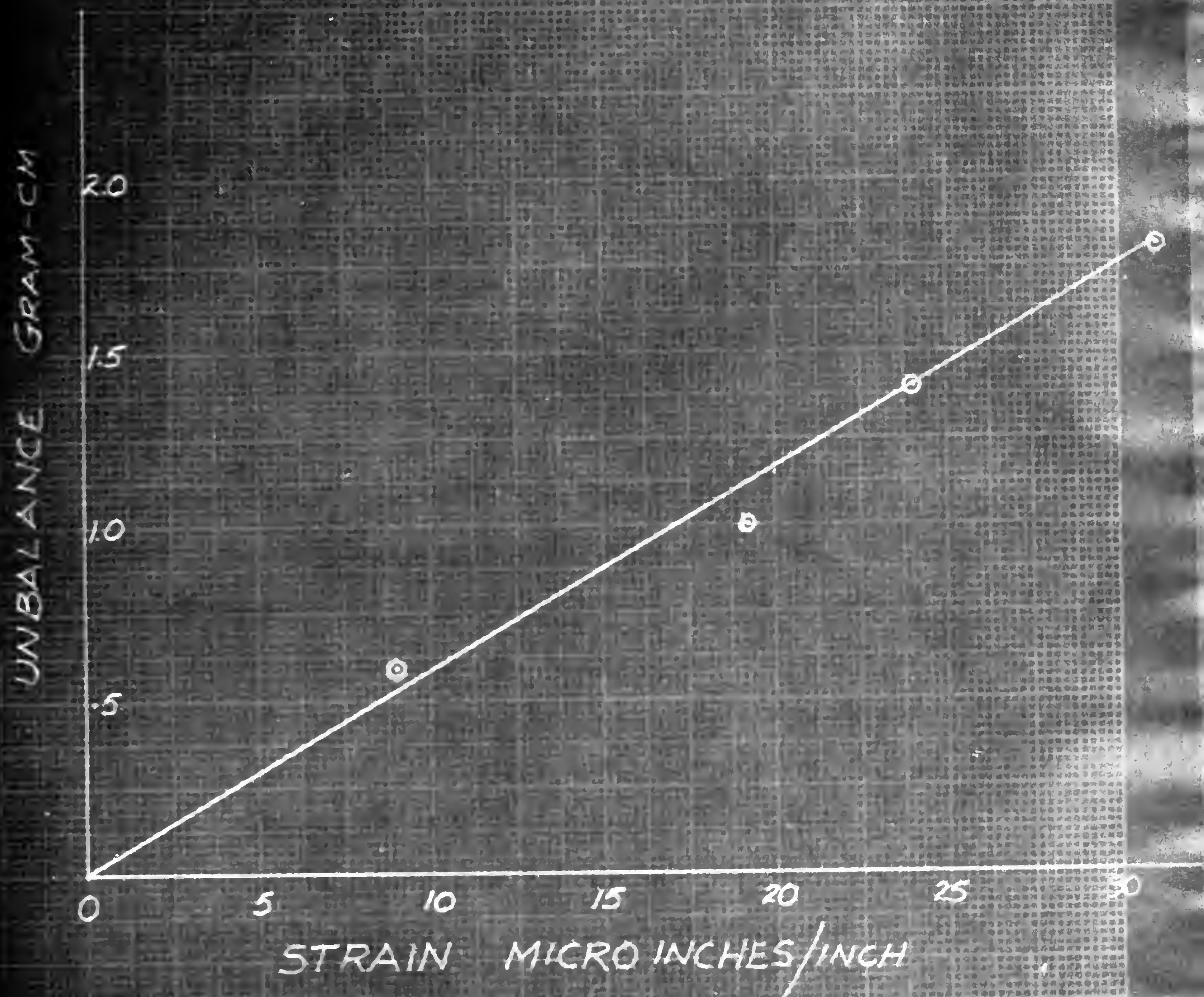
TABLE VI
Beam 1 - Dynamic



DYNAMIC CALIBRATION

BEAM 1

FIG 23



DYNAMIC CALIBRATION

BEAM 2

FIG 24

Unbalance (gram-cm)	Strain(micro inches/inch)
0	0
.594	9
1.005	19.25
1.395	24.0
1.800	32.25

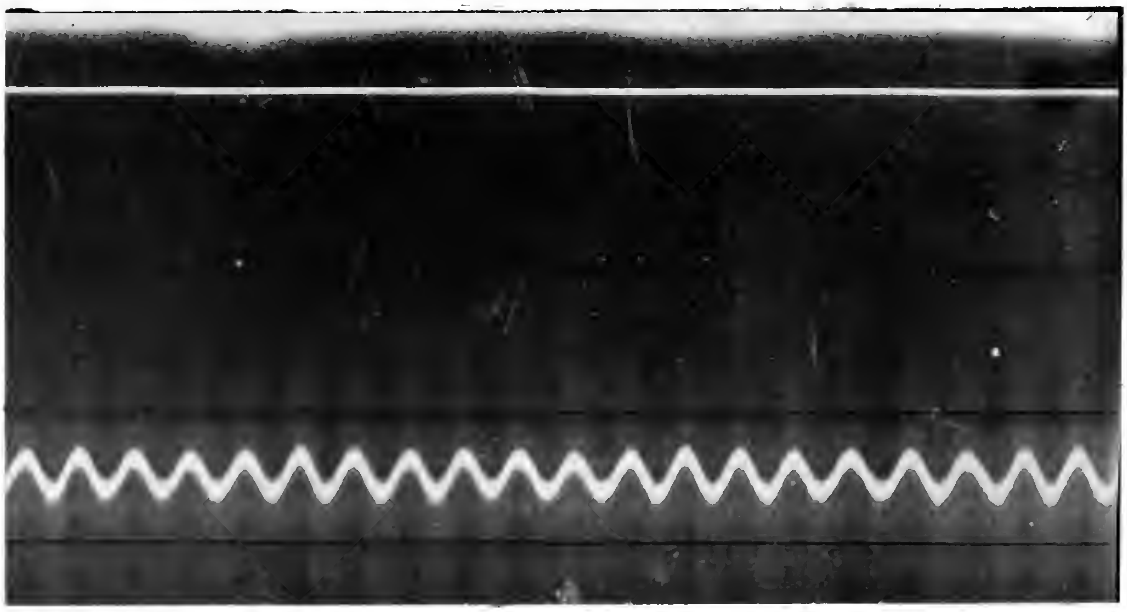
TABLE VII
Beam 2 - Dynamic

For calibration of the machine, the wire strain gages were placed as near as possible to the built-in ends of the beams. The beams were scribed marked for future installation of gages.

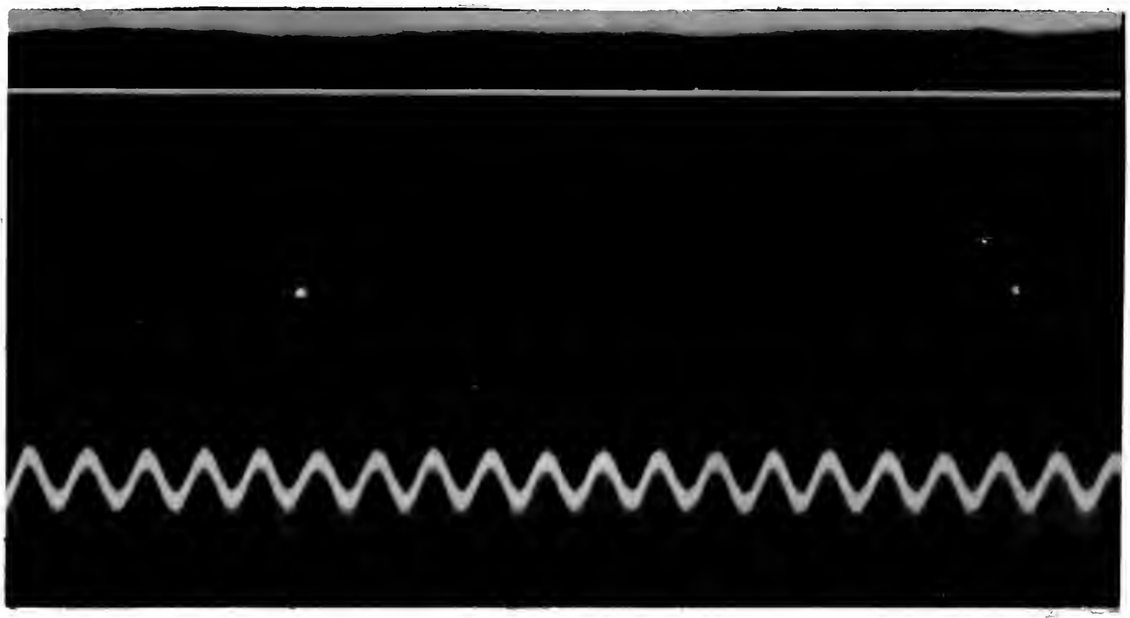
6. Angular location of unbalance.

For very small unbalances it was necessary to use an optical telescope focused on a hairline etched on the beams, to facilitate seeing the movements of the system. When the disk and hairline are stroboscopically illuminated, both the disk and hairline can be made to appear stopped. If the hairline is stopped in one of its maximum positions, the phase angle of the unbalanced weight can be spotted. A telescope was not necessary for the larger unbalanced weights. The unbalance was spotted at the end of its horizontal radius when the beam was in its maximum horizontal position which is in accord with the theory presented by Den Hartog .

OSCILLOGRAPHS

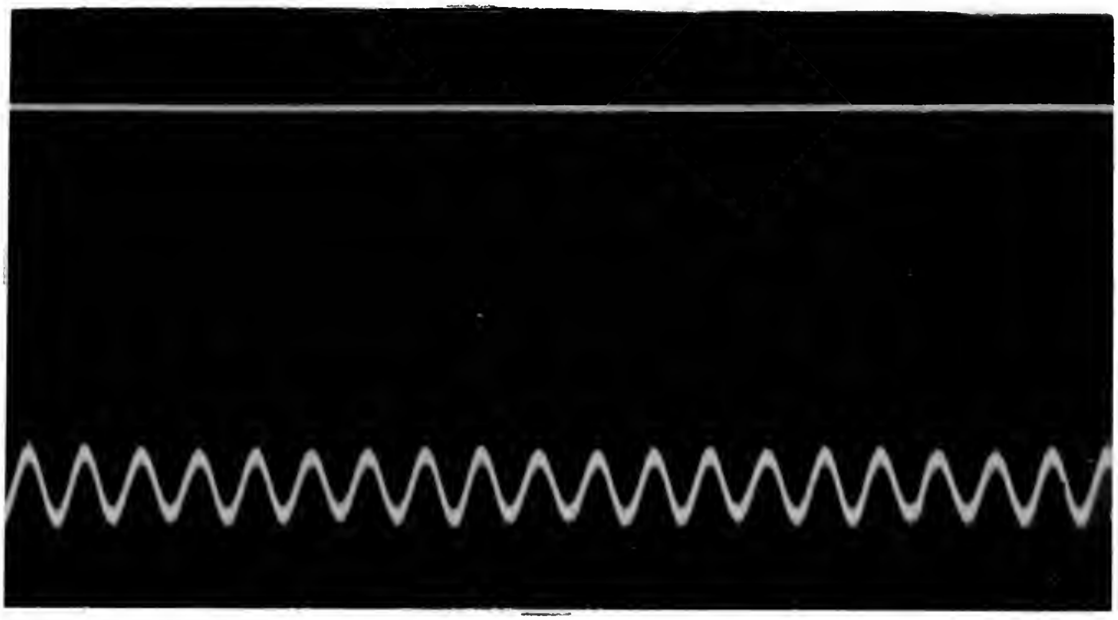


BEAM 1
O UNBALANCE

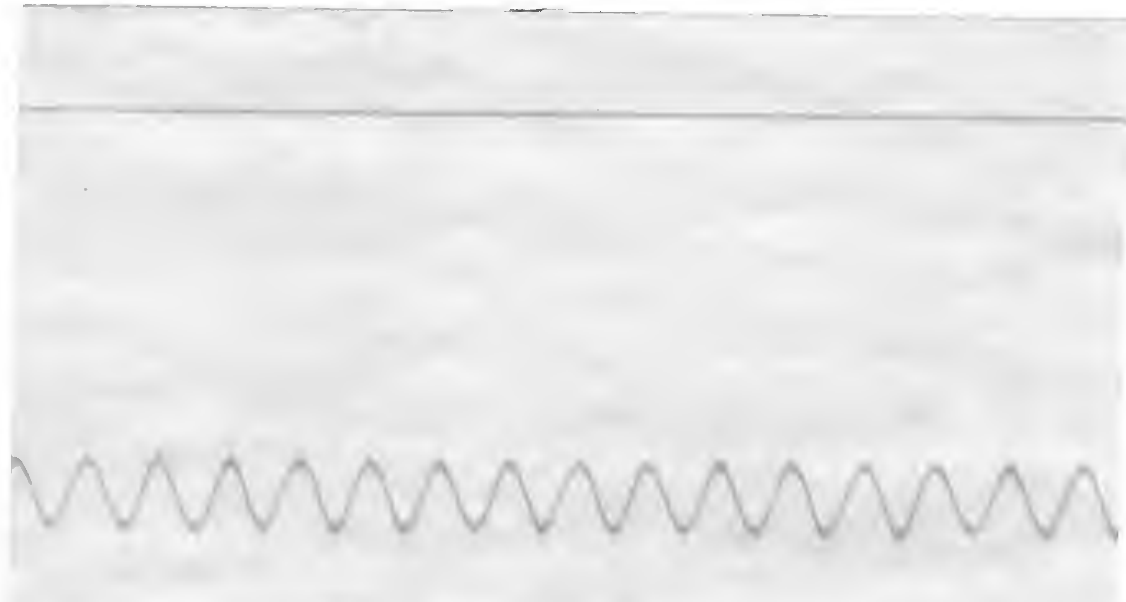


BEAM 2
O UNBALANCE.

OSCILLOGRAPHS



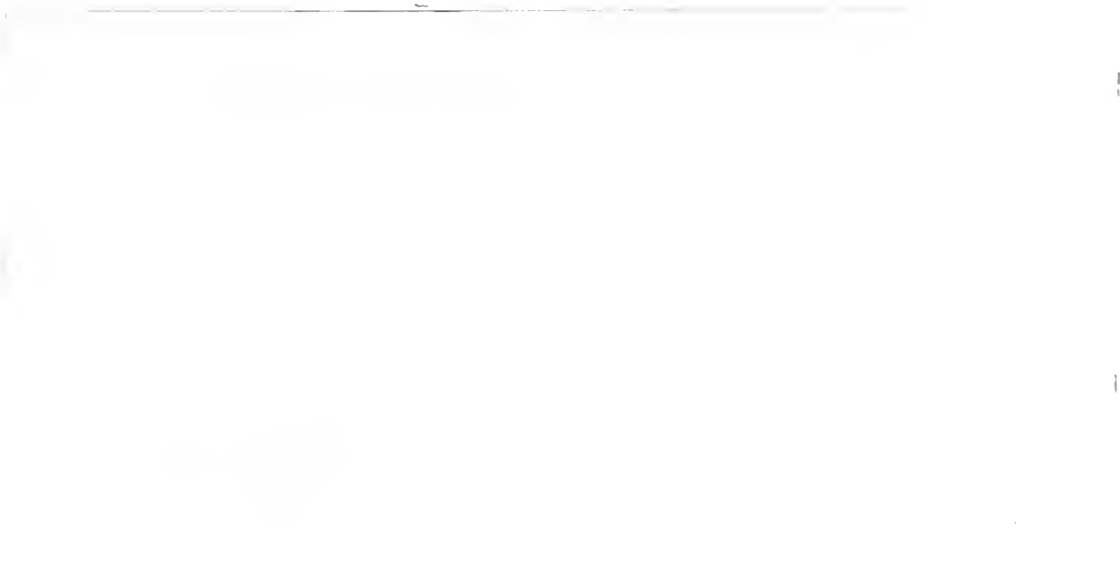
BEAM 1 - STATIC
.156 GRAMS.



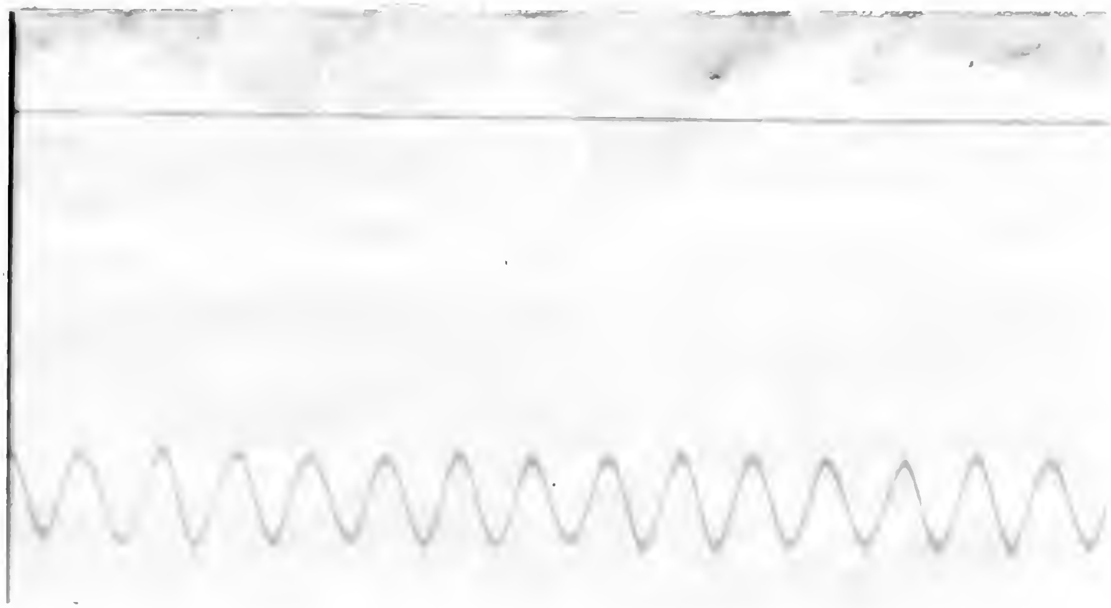
BEAM 2 - STATIC
.156 GRAMS.

Note:- White oscillographs taken with camera of higher speed than Camera used for black oscillographs.

OSCILLOGRAPHS

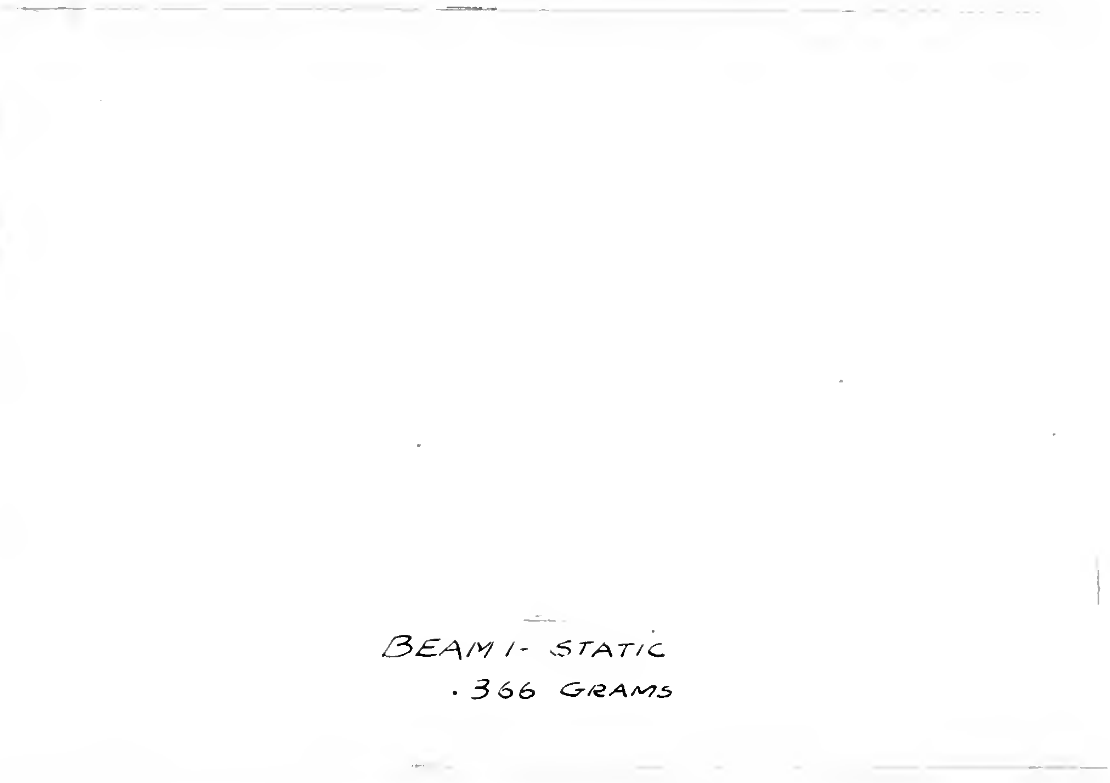


BEAM 1 - STATIC
.264 GRAMS

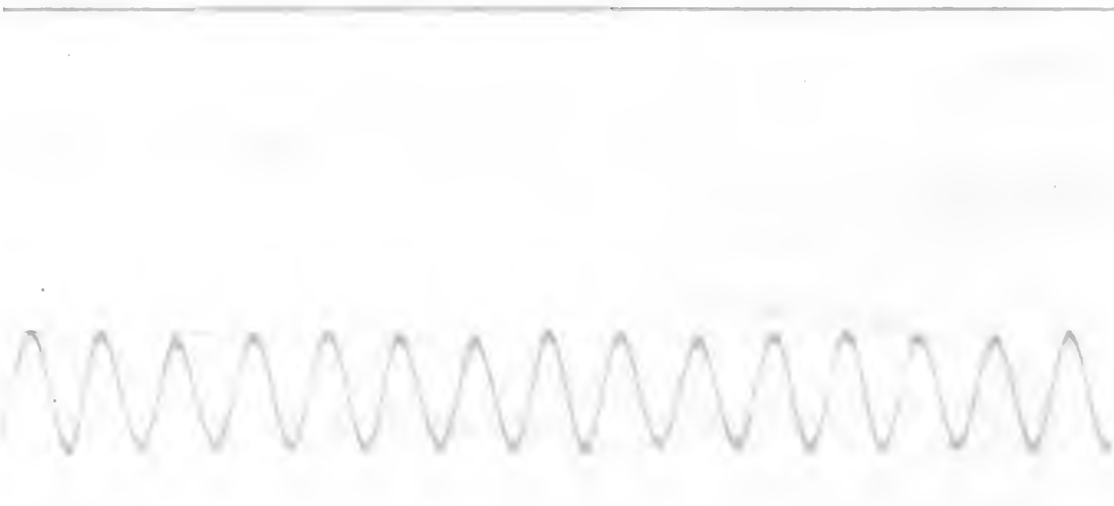


BEAM 2 - STATIC
.264 GRAMS

OSCILLOGRAPHS



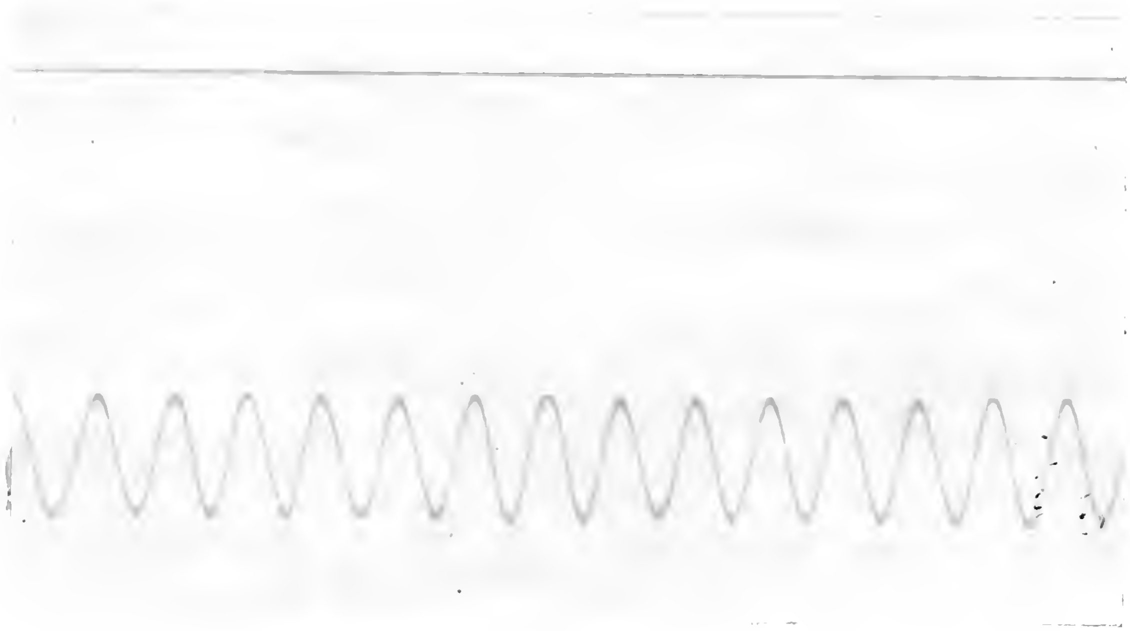
BEAM 1 - STATIC
.366 GRAMS



BEAM 2 - STATIC
.366 GRAMS.

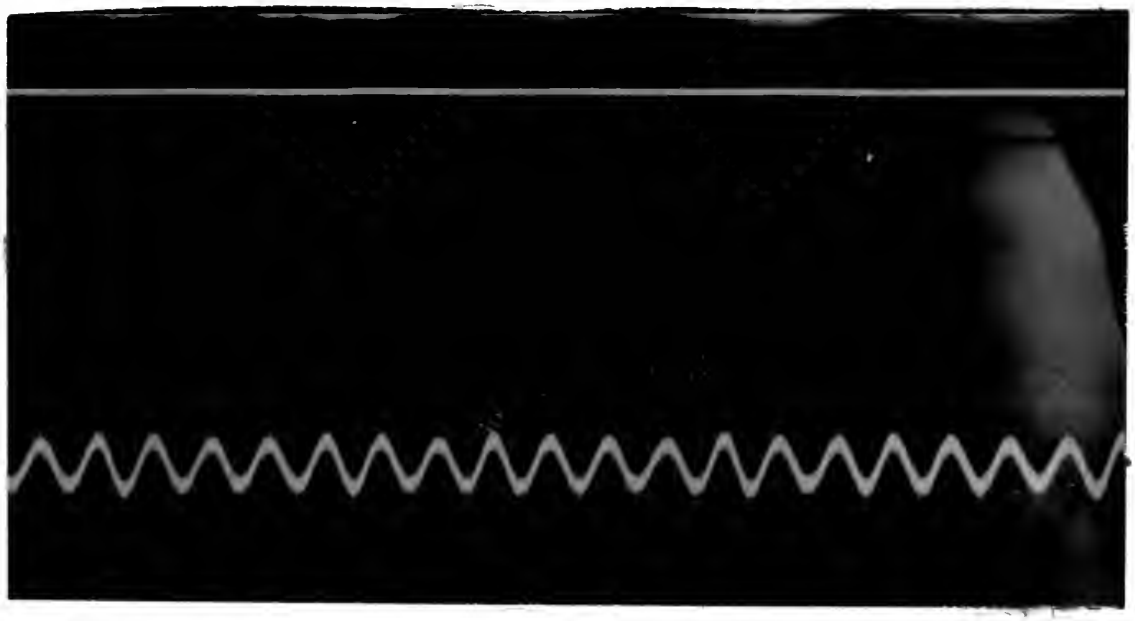
OSCILLOGRAPHS

BEAM 1 - STATIC
.472 GRAMS

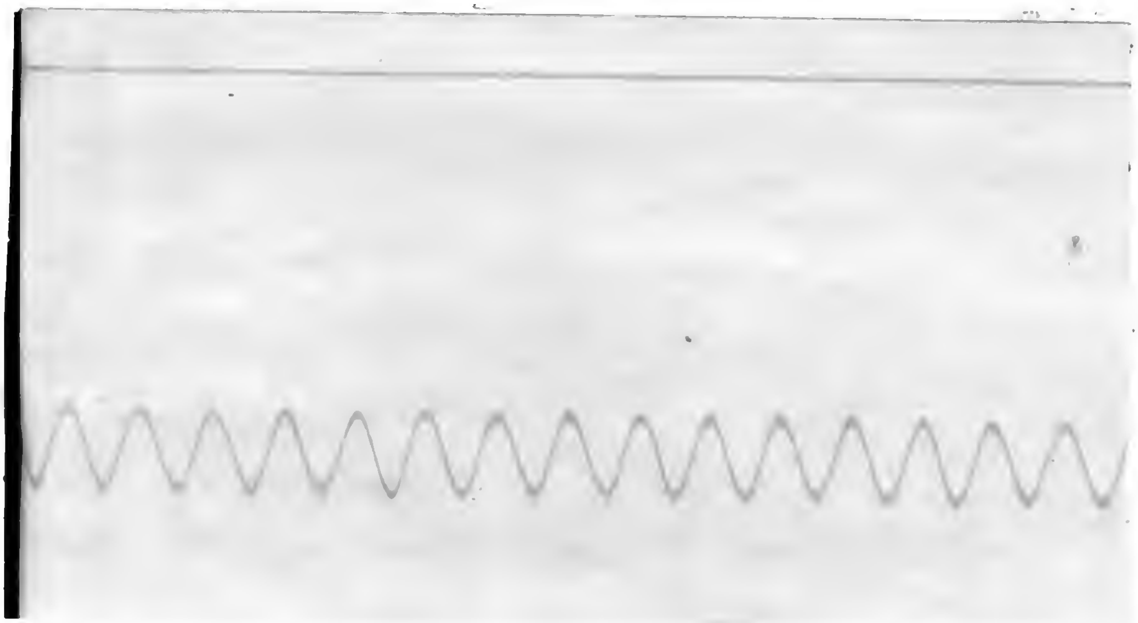


BEAM 2 - STATIC
.472 GRAMS

OSCILLOGRAPHS

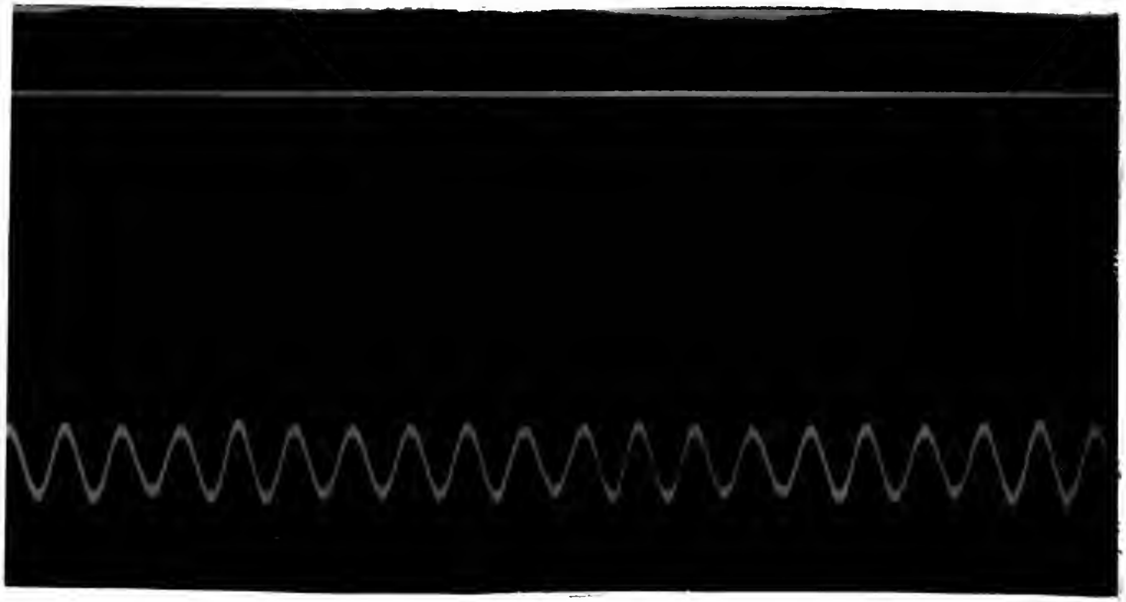


BEAM 1- DYNAMIC
·156 GRAMS

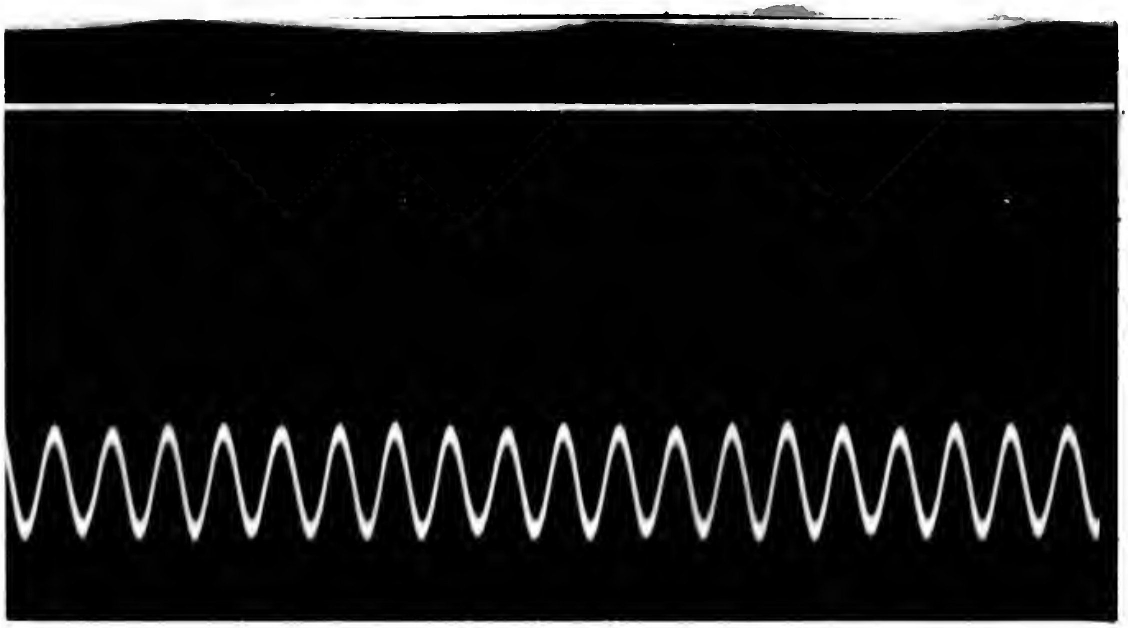


BEAM 2 - DYNAMIC
·156 GRAMS.

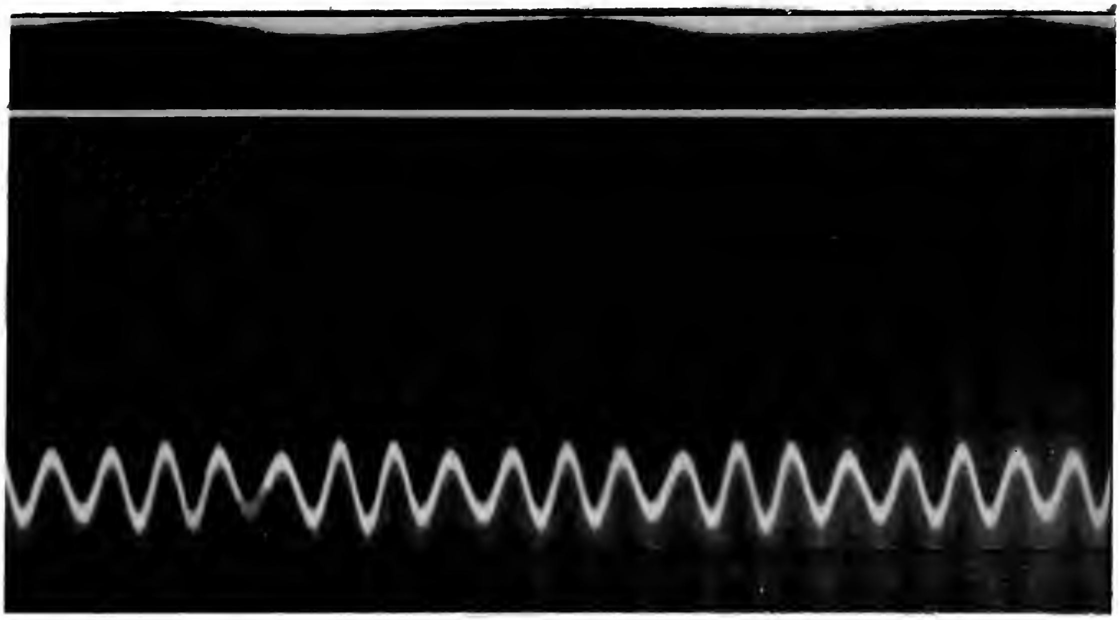
OSCILLOGRAPHS



BEAM 1- DYNAMIC
. 264 GRAMS

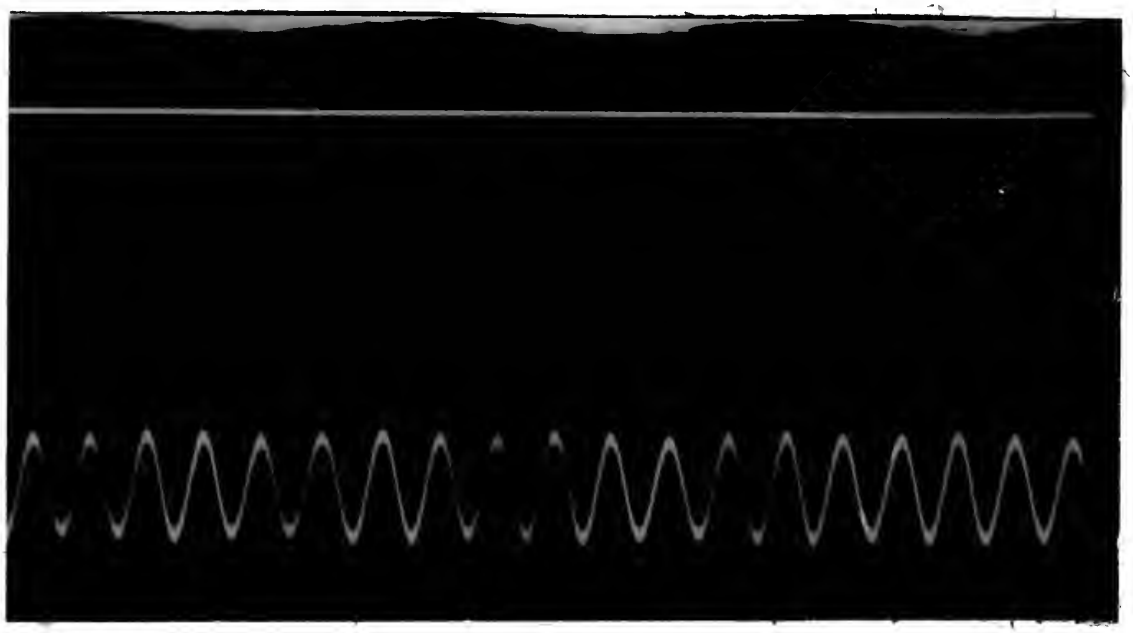


BEAM 2 - DYNAMIC
. 264 GRAMS



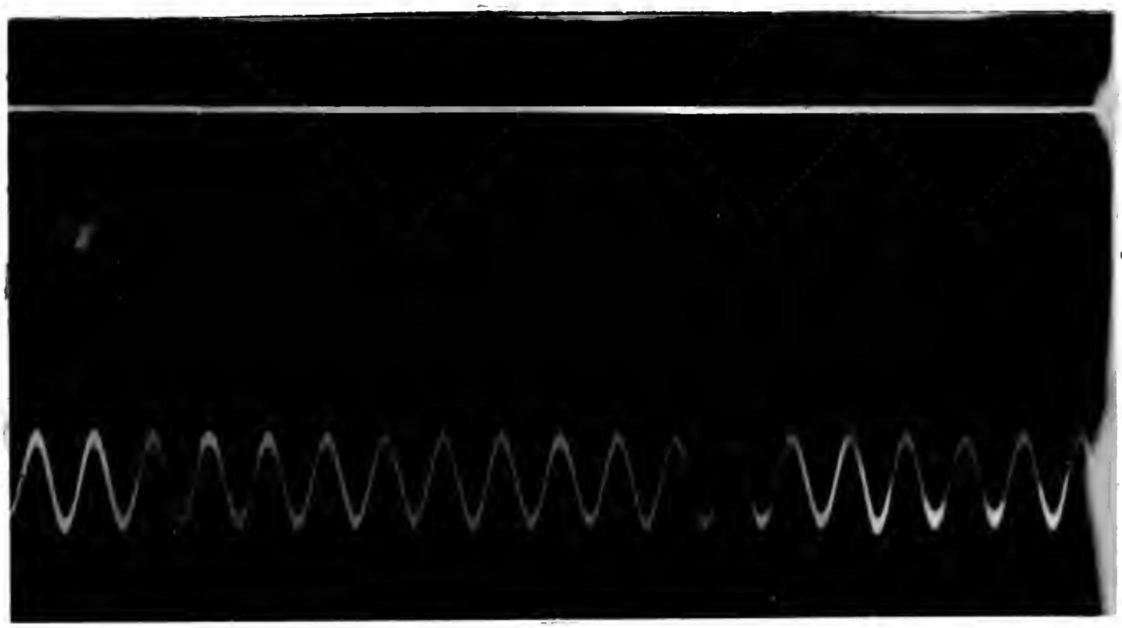
BEAM 1 - DYNAMIC
.366 GRAMS

FOR This run - extraneous vibrations occured. Probably due to bearings.

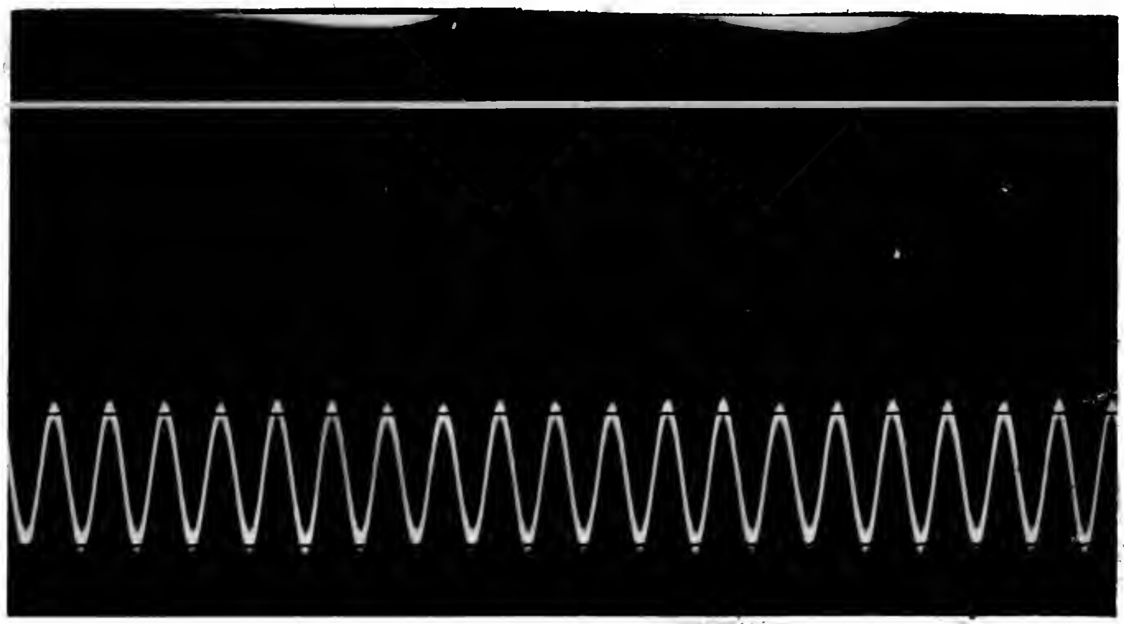


BEAM 2 - DYNAMIC
.366 GRAMS.

OSCILLOGRAPHS



BEAM 1 - DYNAMIC
.472 GRAMS



BEAM 2 - DYNAMIC
.472 GRAMS.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

The curves for both static and dynamic unbalance indicate the machine is quite sensitive. The smallest unbalance detected was .594 gm-cm . but it is believed that if the test shaft had been more perfectly balanced smaller unbalances could be detected.

The machine is in the stage of completion such that a rotor with unknown unbalance could be balanced very nearly perfect. The method of procedure would of course be by trial and error which is time consuming. It necessitates the addition of various size weights at different angular locations in two predetermined planes. The rotor will be completely balanced when no wave form and therefore no vibration is recorded. The calibration curves would serve to give the approximate magnitude of unbalance but would not predict the location along the axis.

To make the method more automatic, a procedure must be devised to separate the vibrations due to static unbalance from those due to dynamic unbalance. This would consist of an electrical network capable of adding and subtracting impulses. The author did not investigate this phase of the project, but further work by members of the Mechanical Engineering Department is contemplated.

During the course of the experiment several faults were noticed which had a detrimental effect on the results. Most of these can be contributed to faulty design. The results show

that one beam has a 5% greater spring constant than the other. This difference shows up in the slopes at the calibration curves. The difference can be contributed to variation in dimensions and hardness of the material.

After prolonged vibration slight misalignment of the wheels occurred which prevented smooth rotation of the rotor, and slight clearances developed where the relatively soft aluminum was fitted into steel.

For optimum results, steel bushings should be sweated on the aluminum members which fit snugly into the steel base and on the aluminum wheel shafts, which project into the inside race of the ball bearings.

Although very small precision ball bearings were used in a preloaded condition, excess clearance was still prevalent. Timken tapered bearings would eliminate this to some extent. To eliminate extraneous vibrations due to bearings, no matter what type used, would necessitate eliminating the bearings completely.

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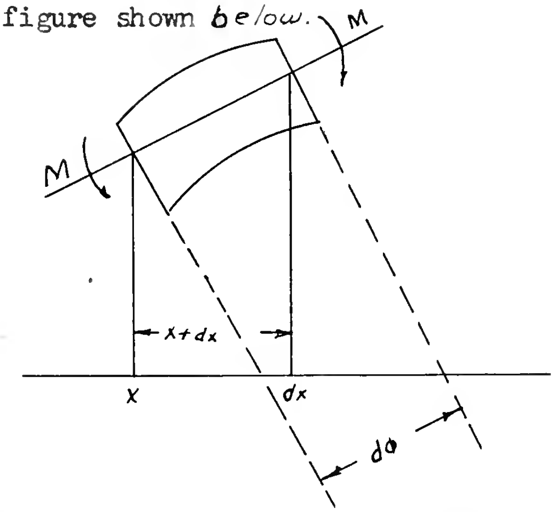
APPENDIX

Derivation for the Potential energy of a cantilever beam.

Consider the figure shown below.



in which a small element of the beam is under the influence of the bending moment M. The element is originally straight and is bent through an angle $d\phi$ by the moment M. If the left-hand end of the element is assumed to be clamped, the moment M at the right hand end turns through the angle $d\phi$. The work done by M on the beam is 1/2 $M\,d\phi$ where the factor 1/2 appears because M and $d\phi$ are increasing from zero together. This work is stored as potential energy in the beam element.

Now calculate the angle $d\phi$. If the slope at the left-hand end x be $\frac{dy}{dx}$, then the slope at the right-hand end is $\frac{dy}{dx} + \left(\frac{d^2y}{dx^2}\right) dx$ and the difference in slope $d\phi$ is

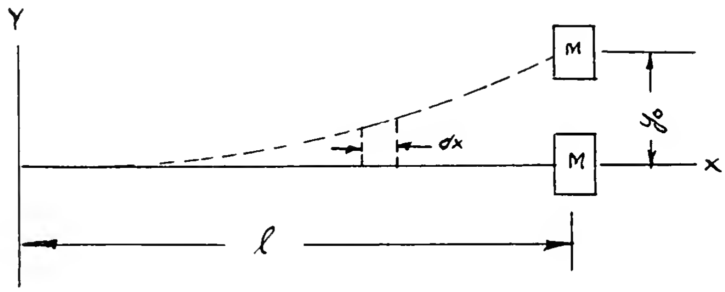
$$d\phi = \frac{d^2y}{dx^2}\, dx$$

so that $d\,pot = \frac{1}{2} M y'' dx.$

With the differential equation of bending $M = EIy''$

then $$Pot. = \frac{EI}{2}\int_0^{\ell}\left(\frac{d^2y}{dx^2}\right)^2 dx$$

The kinetic energy for a cantilever beam with a concentrated mass can be derived as follows:



Consider an element (dx) neglecting the concentrated mass.

The Kinetic energy = $\frac{1}{2} m v^2$

For an element~~ed~~al section KE = $\frac{1}{2}(\mu_1 dx)(y\omega)^2$

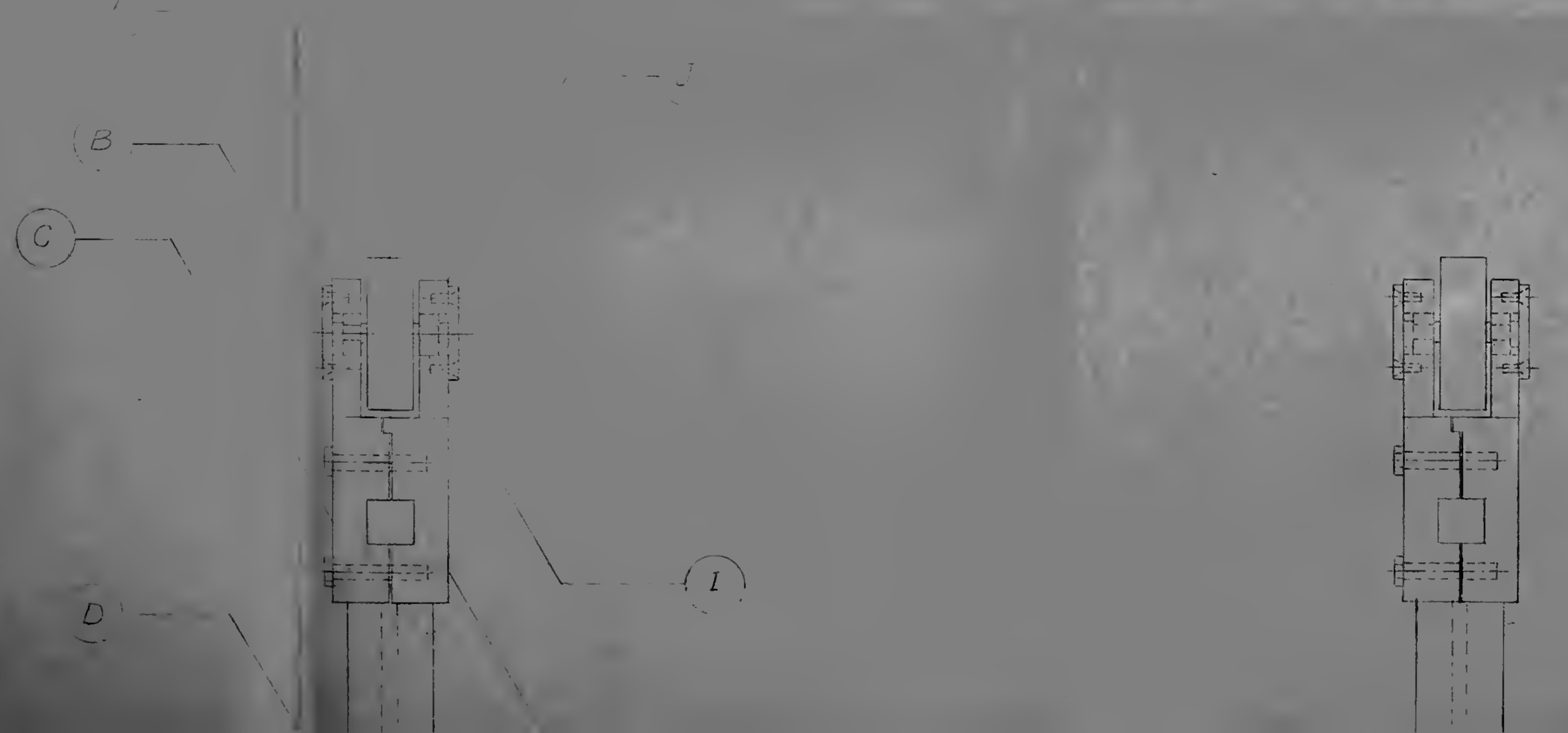
and the total Energy = $\frac{1}{2}\mu_1\omega^2\int_0^{\ell} y^2 dx$

Now consider a massless beam loaded with a concentrated mass.

The Kinetic Energy = $\frac{1}{2} M y_0^2 \omega^2$

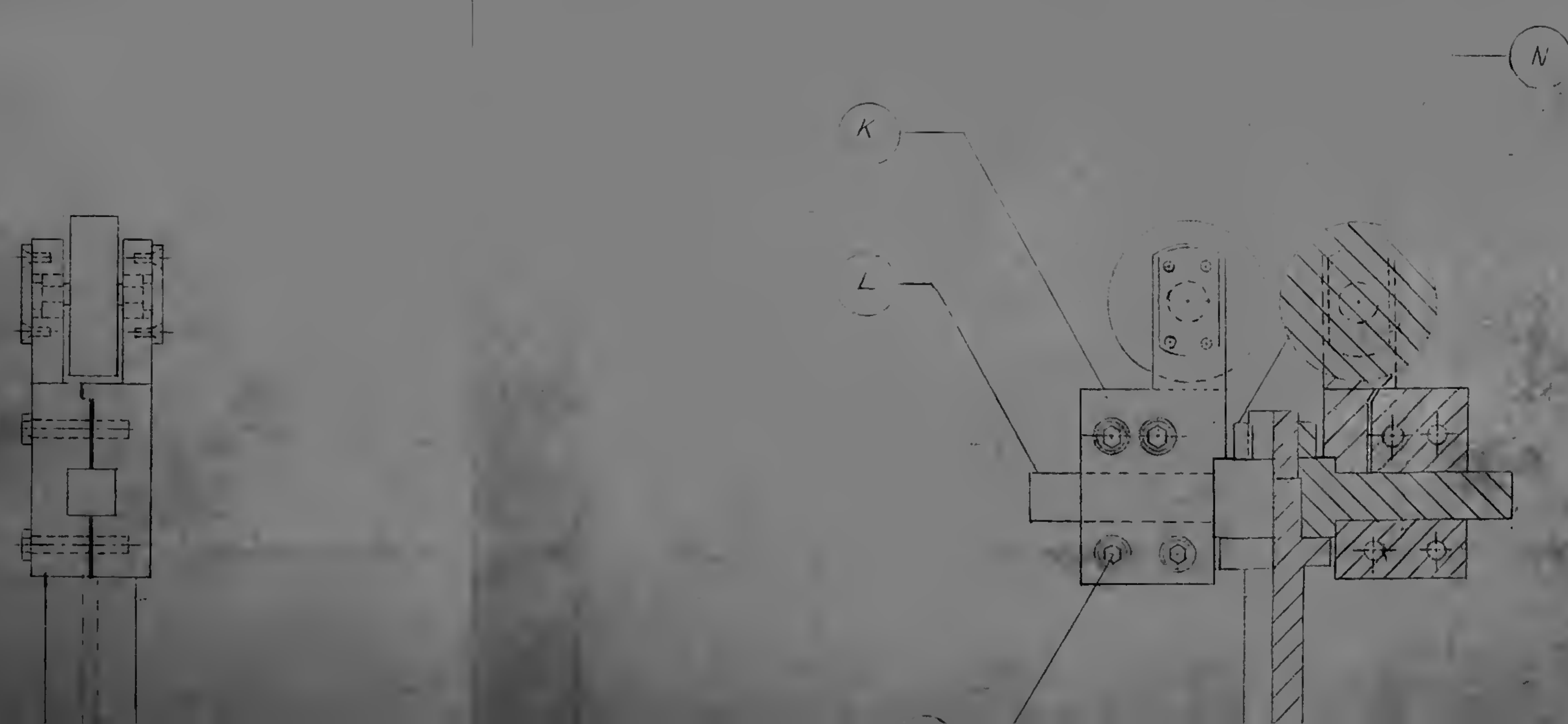
Kinetic energy of system = $\frac{1}{2}\mu_1\omega^2\int_0^{\ell} y^2 dx + \frac{1}{2} M y_0^2 \omega^2$

Thesis 13117
H17 Hammerstone
Design and test of a dynamic balancing machine for small rotors.

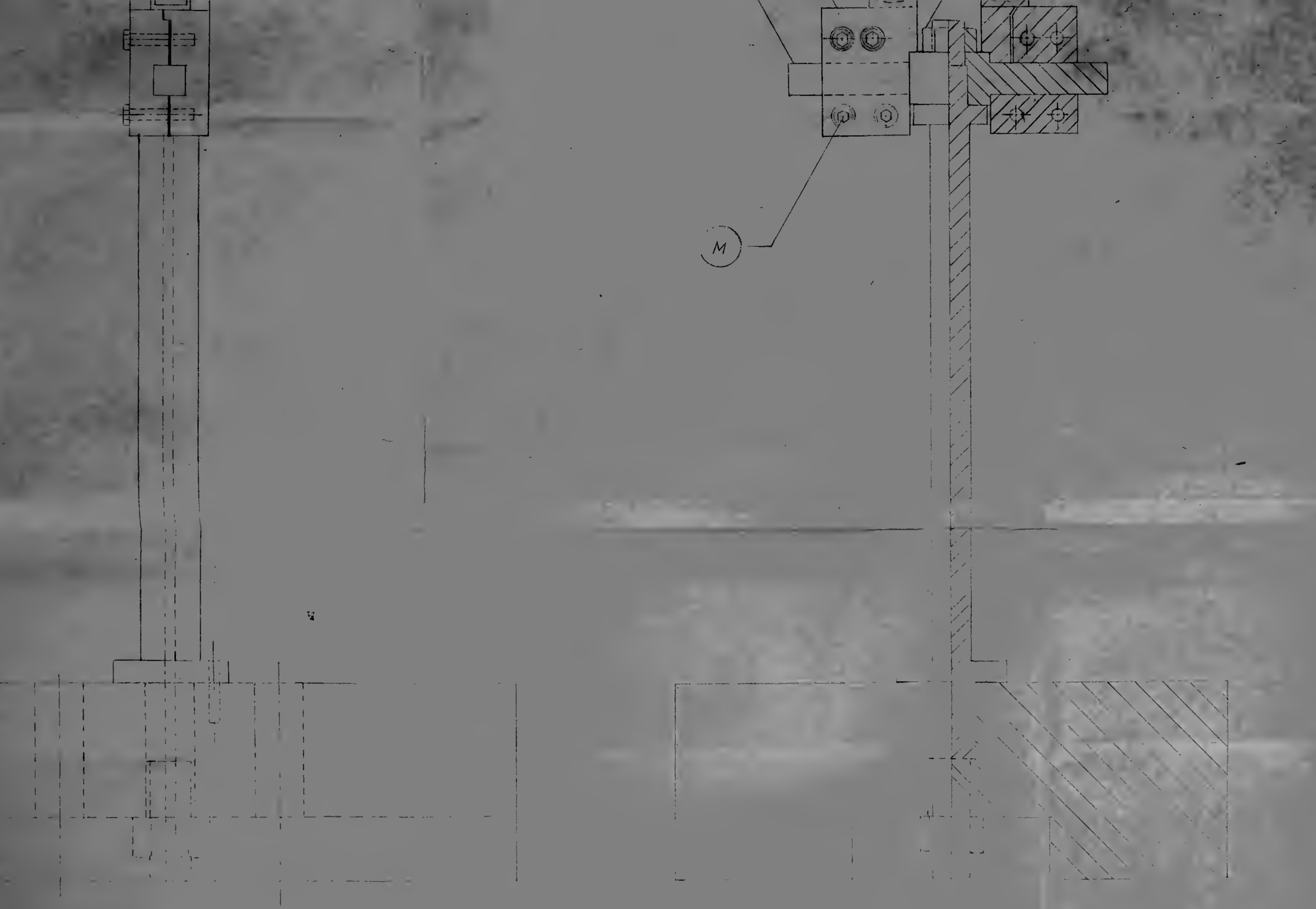

B
C
D
I
J

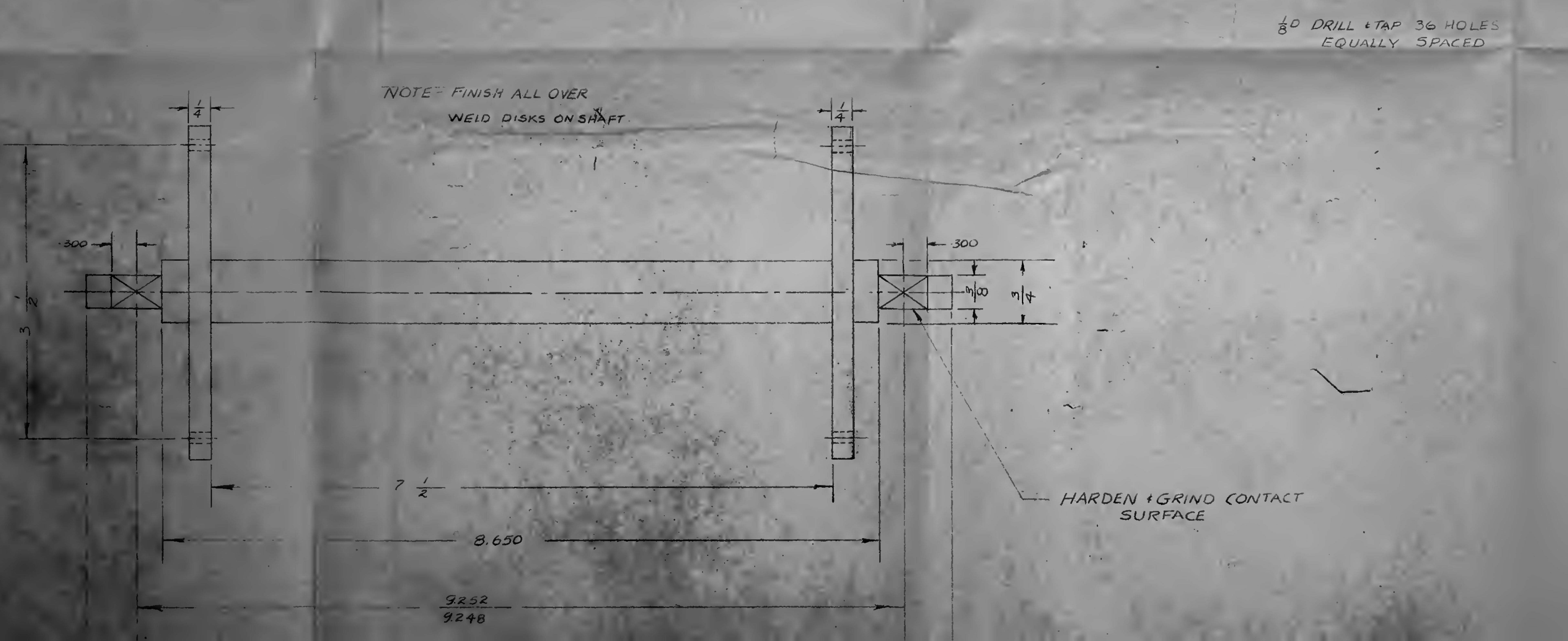
BILL OF MATERIALS

No	NAME	Q	MATERIAL	REMARKS
A	WHEEL	4	DURALUMINUM	
B	BEARING COVER PLATE	8	DITTO	
C	RIGHT FRONT WHEEL BRKT	2	DITTO	
L	SUPPORT BEAM	2	DITTO	
E	½" DIA A.S HEAVY NUT-.504" THICK SEMI FINISHED	4	HEX-SEMI-FINISH.	PURCHASED
F	BASE	1	MILD CARBON STL	
G	DOWEL PIN	2	DURALUMINUM	
H	RIGHT BACK WHEEL BRKT	2	DITTO	
I	BALL BEARING	8	NEW DEPARTURE LIGHT SERIES	PLAIN R-2 PURCHASE
J	FLAT HEAD MACH SCREW	32	A.S. NOMINAL SIZE 2 LENGTH	PURCHASE
K	LEFT FRONT WHEEL BRKT	2	DURALUMINUM	
L	CROSS SUPPORT	2	DITTO	
M	HEX SOCKET TYPE CAP SCREW A.S NOMINAL 8	32		PURCHASE
N	⅜" DIA A.S REG HEX NUT	2	.295" THICK-SEMI FINISHED	PURCHASE

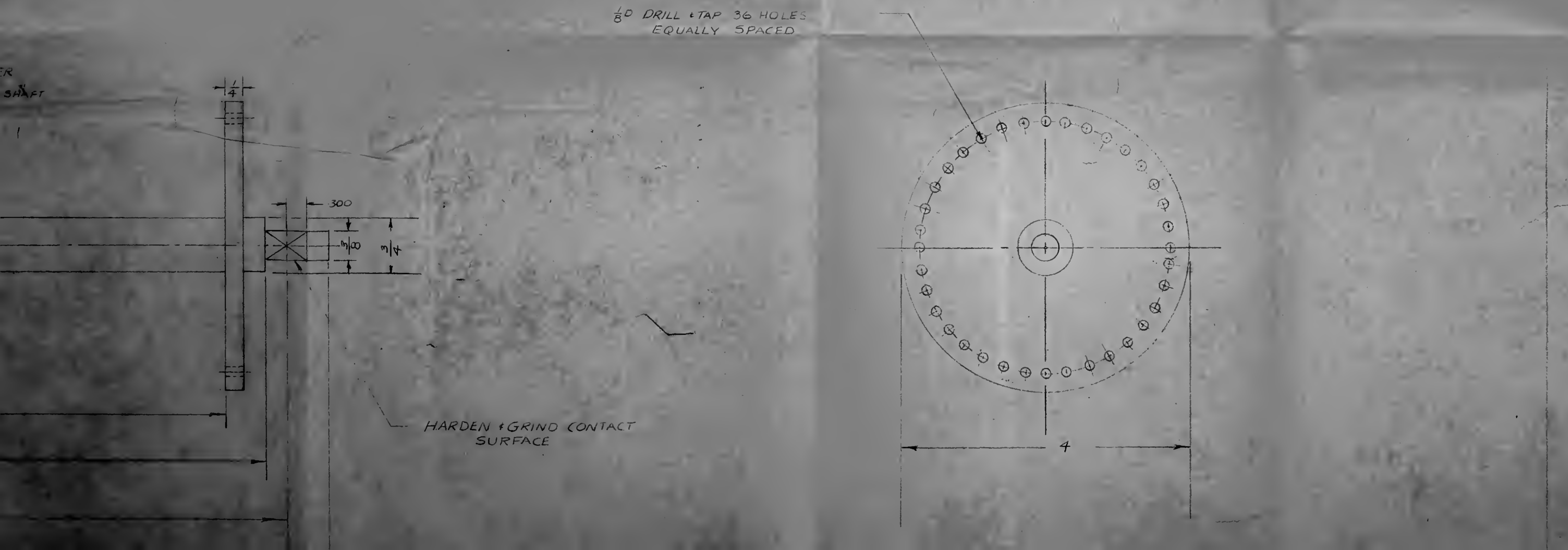



I
D
H
G
F
M

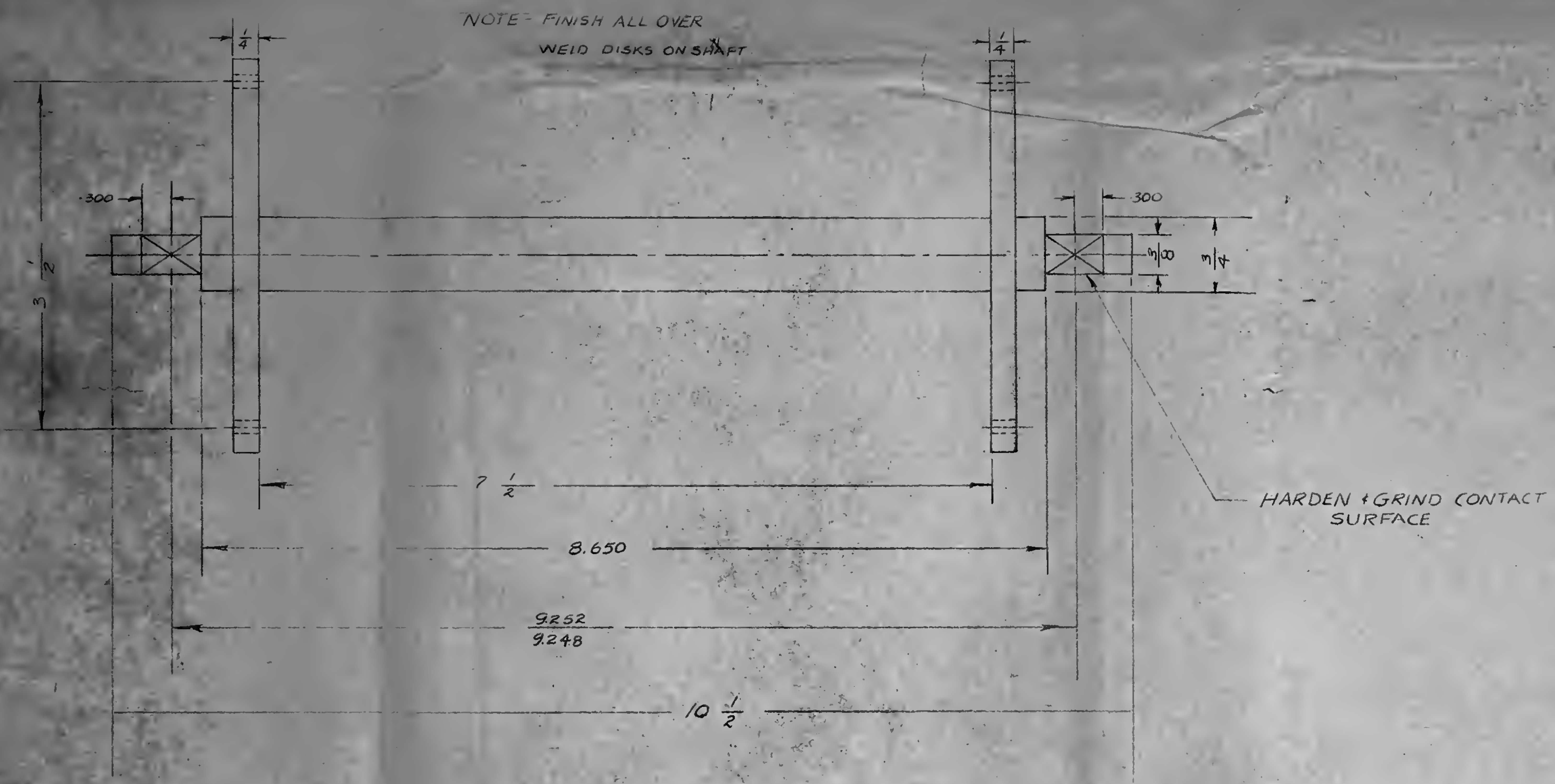

M



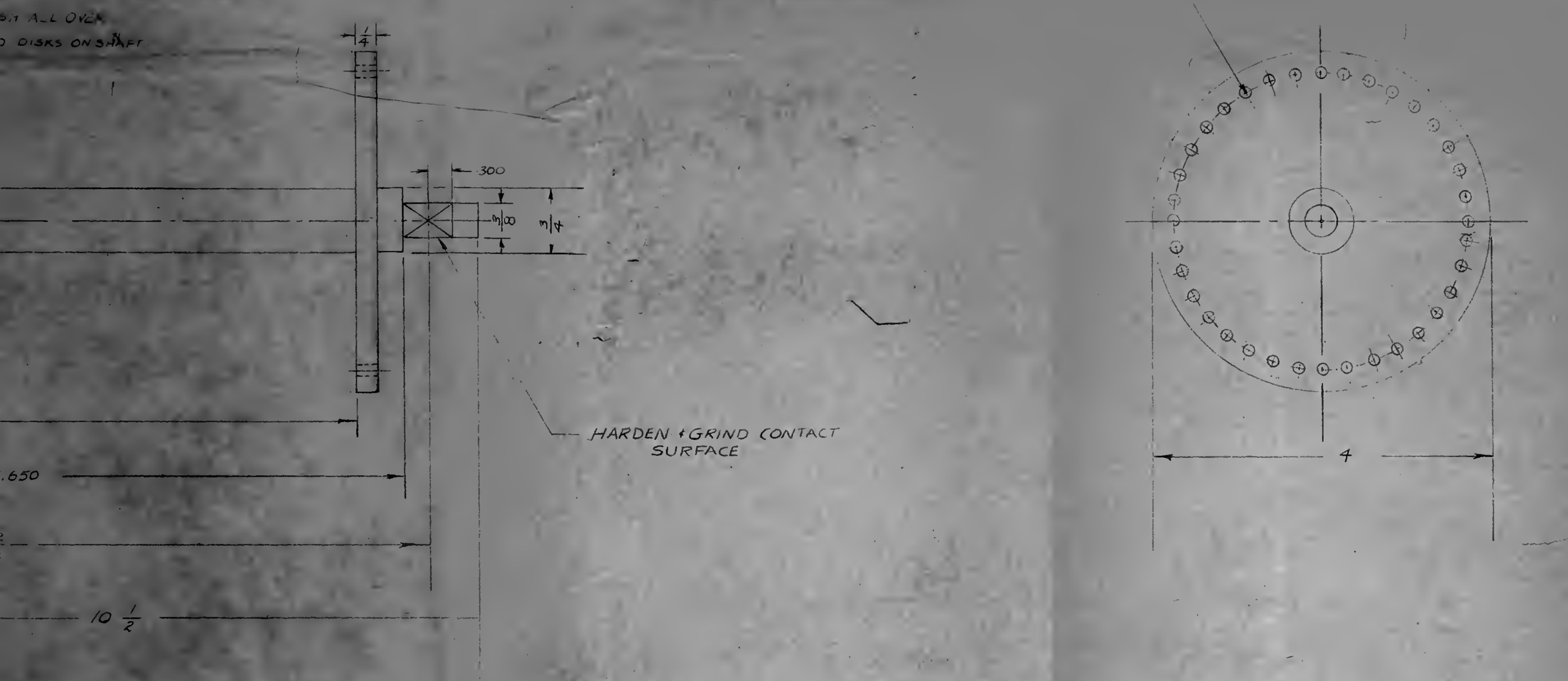
1/8 D DRILL & TAP 36 HOLES
EQUALLY SPACED
NOTE- FINISH ALL OVER
WELD DISKS ON SHAFT
1/4
1/4
.300
.300
3 1/2
3/8
3/4
7 1/2
8.650
9.252
9.248
HARDEN & GRIND CONTACT
SURFACE



1/8 D DRILL & TAP 36 HOLES
EQUALLY SPACED
SHAFT
1/4
300
3/8
3/4
HARDEN & GRIND CONTACT
SURFACE
4


NOTE- FINISH ALL OVER
WELD DISKS ON SHAFT
1/4
1/4
.300
.300
3 1/2
3/8
3/4
HARDEN & GRIND CONTACT SURFACE
7 1/2
8.650
9.252
9.248
10 1/2

TEST SHAFT



TEST SHAFT

TEST SHAFT

1 PC REQUIRED

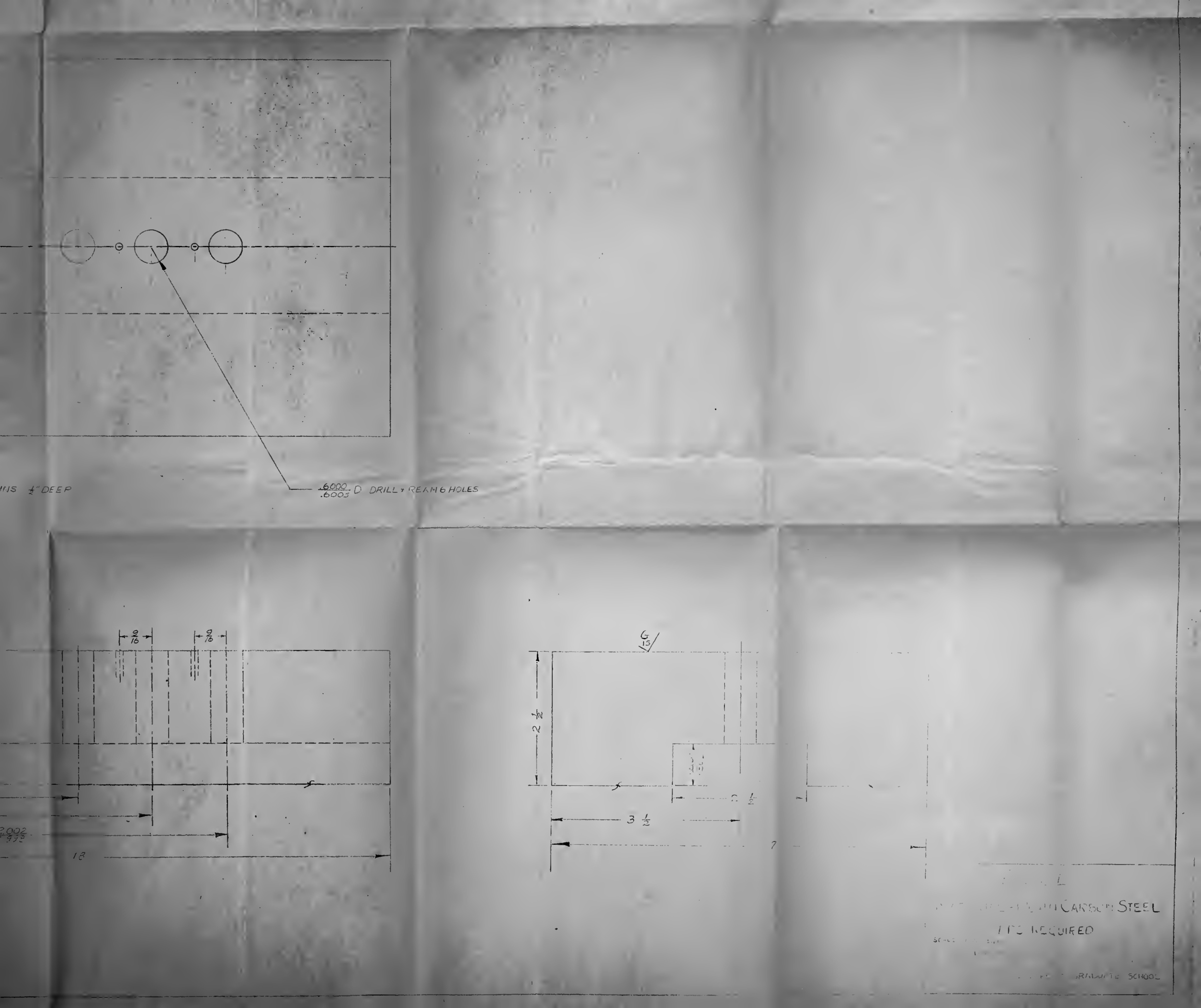
MATERIAL - PLAIN CARBON STEEL

SCALE - FULL SIZE

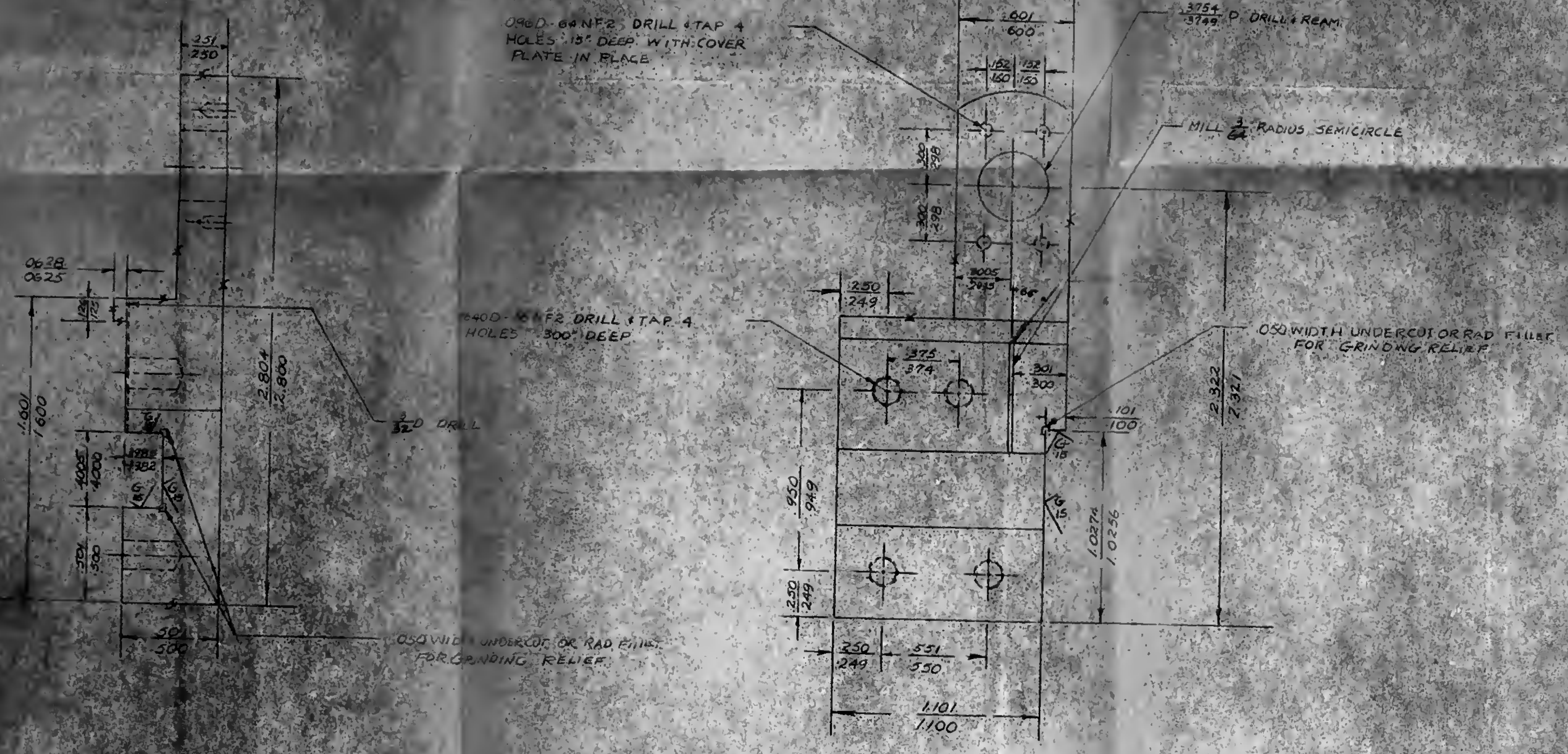
J.E. HAMMERSTONE

U.S.N. POST GRADUATE SCHOOL

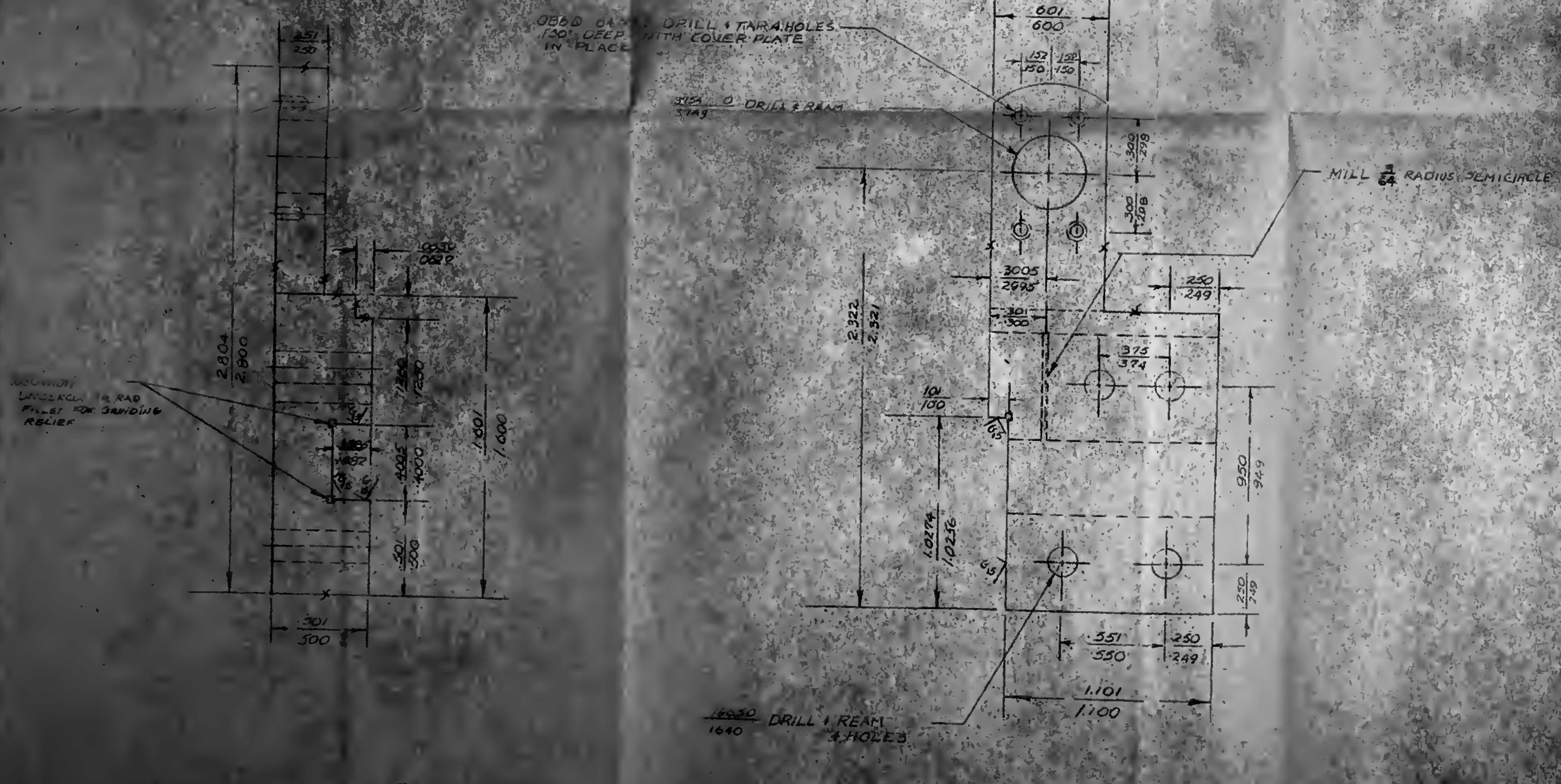

3 1/2
7
1/8 D DRILL IN ASSEMBLY FOR DOWEL PINS 1/4" DEEP
.6000/.6003 D DRILL + REAM 6 HOLES
9/16
G 15
6.502/6.498
9.252/9.248
12.002/11.998
18.



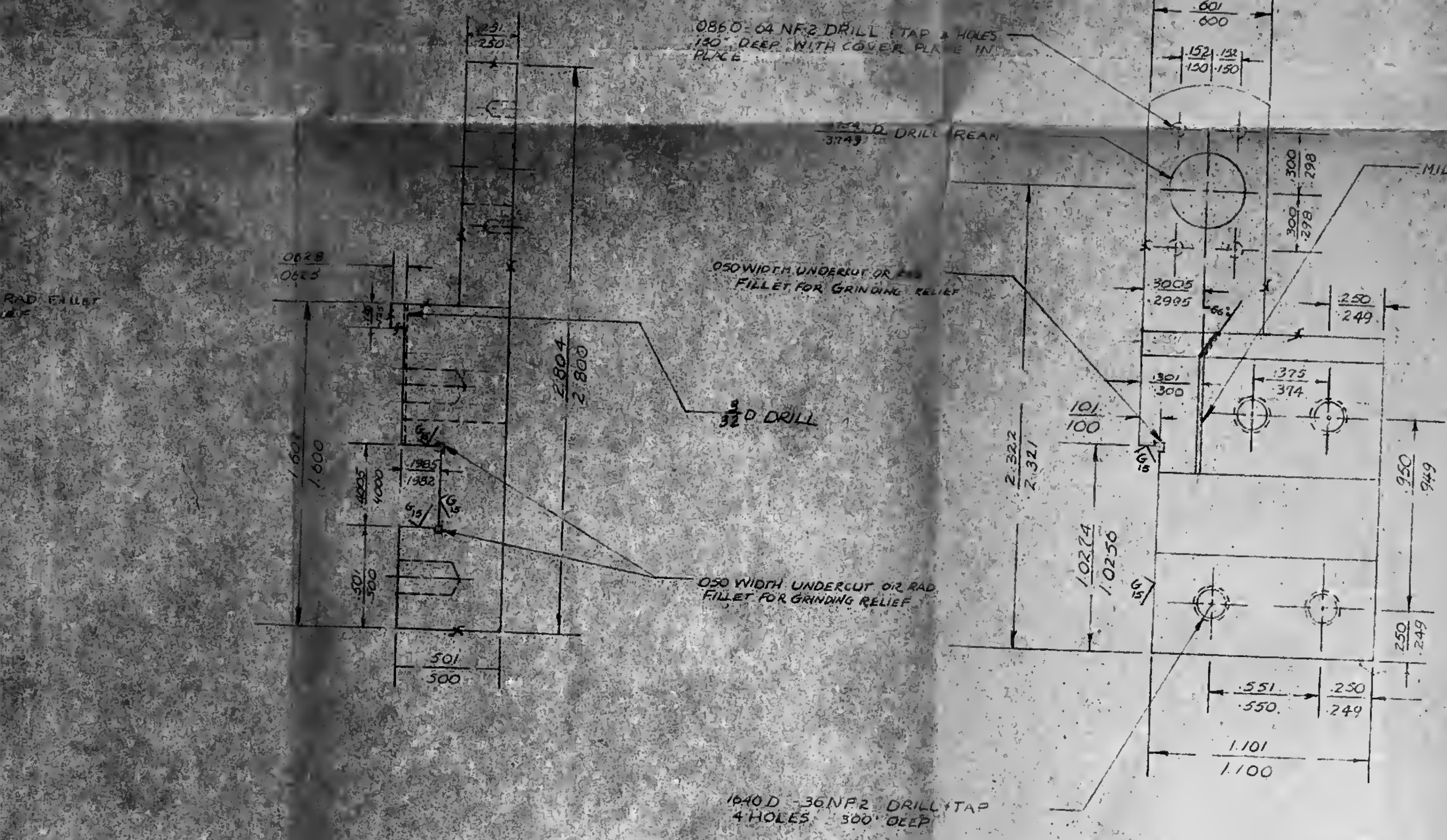
½" DEEP
.6000 / .6005 D DRILL + REAM 6 HOLES
9/16
9/16
18
2 ½
3 ½
7
CARBON STEEL
REQUIRED
GRADUATE SCHOOL



BACK HALF LEFT WHEEL BRACKET



FRONT HALF RIGHT WHEEL BRACKET



BACK HALF RIGHT WHEEL BRACKET

BACK HALF LEFT WHEEL BRACKET
2 REQUIRED
MATERIAL DURALUMINUM

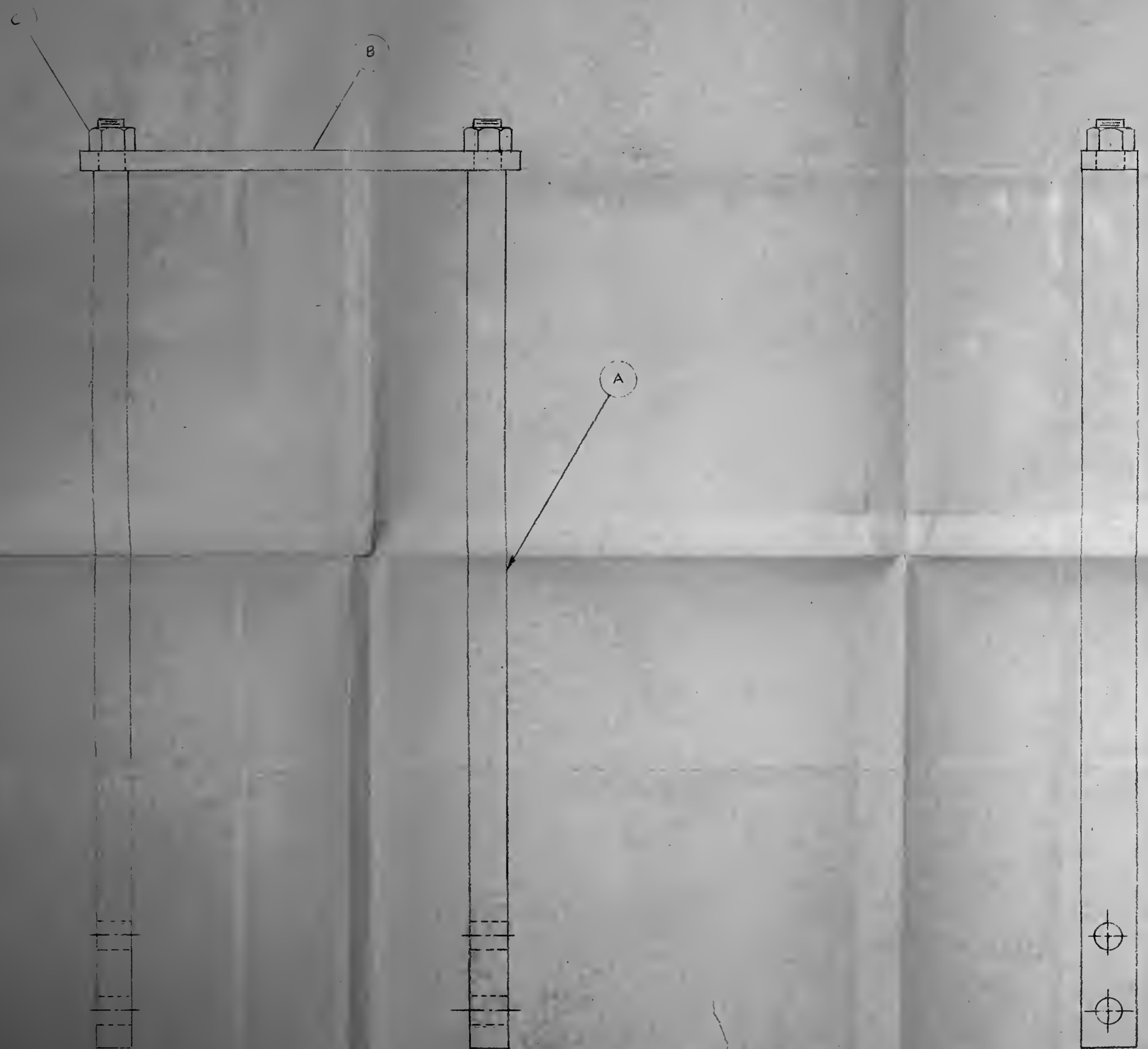
FRONT HALF RIGHT WHEEL BRACKET
2 REQUIRED
MATERIAL DURALUMINUM

BACK HALF RIGHT WHEEL BRACKET
2 REQUIRED
MATERIAL DURALUMINUM

SCALE DOUBLE SIZE

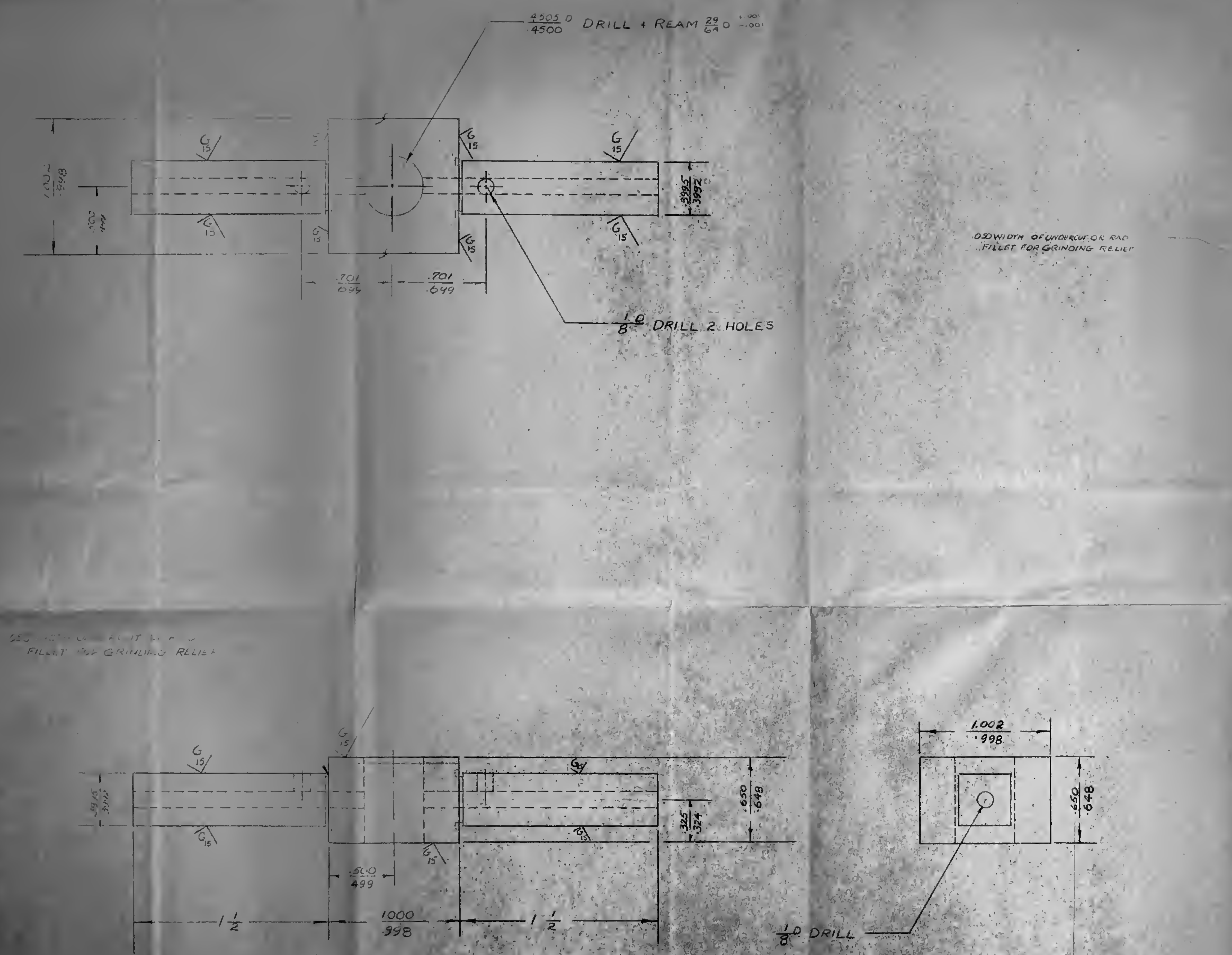
J.E. HAMMERSTONE

U.S.N. POSTGRADUATE SCHOOL



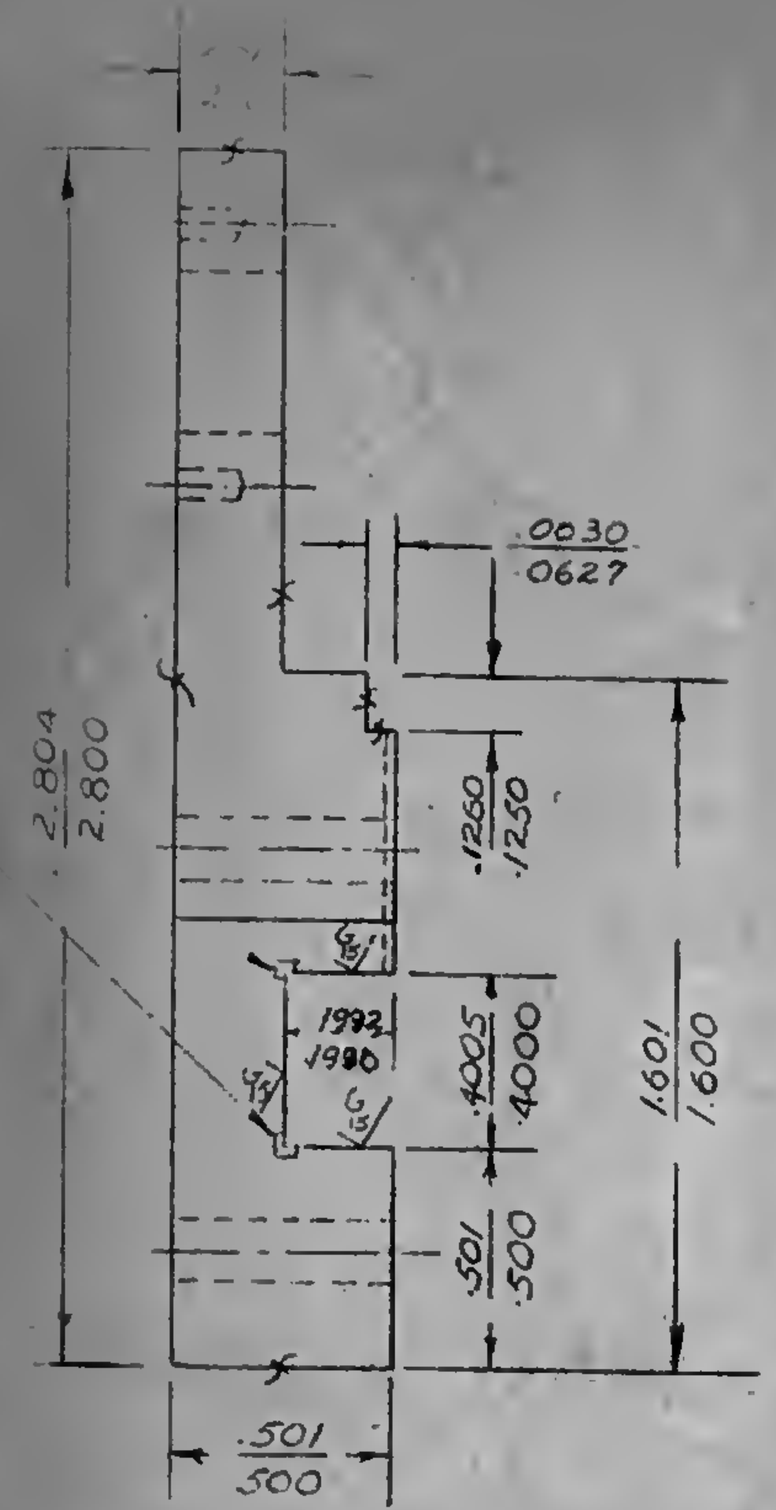
PC NO	NAME	Q	MATERIAL
	BILL OF MATERIALS		
A	UPRIGHT	4	PLAIN STEEL
B	CROSS PIECE	2	PLAIN STEEL
C	3/8 A.S NUT	4	PURCHASE

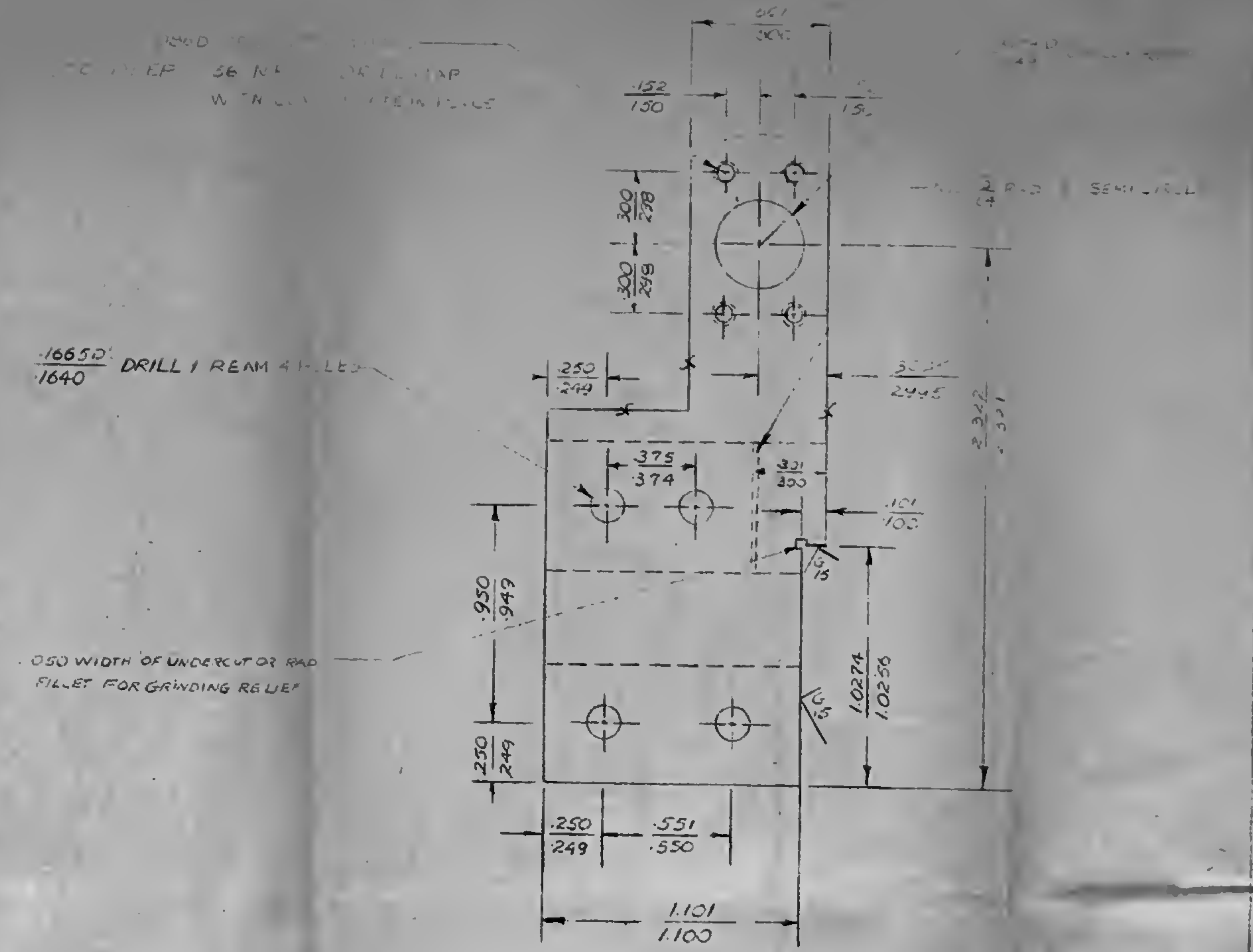
J.E. HAMMER 1942



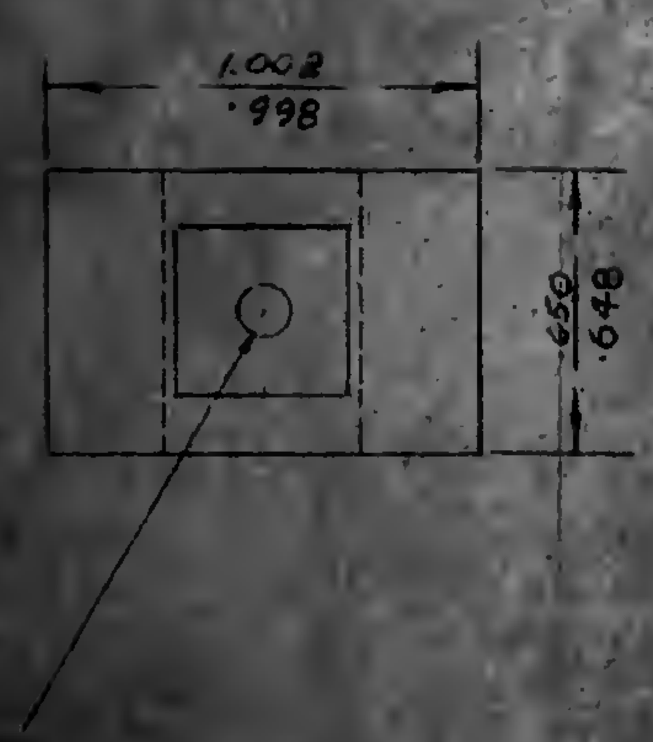
CROSS SUPPORT

.050 WIDTH OF UNDERCUT OR RAD. FILLET FOR GRINDING RELIEF





FRONT HALF LEFT WHEEL BRACKET



CROSS SUPPORT

2 REQUIRED

MATERIAL - DURALUMINUM

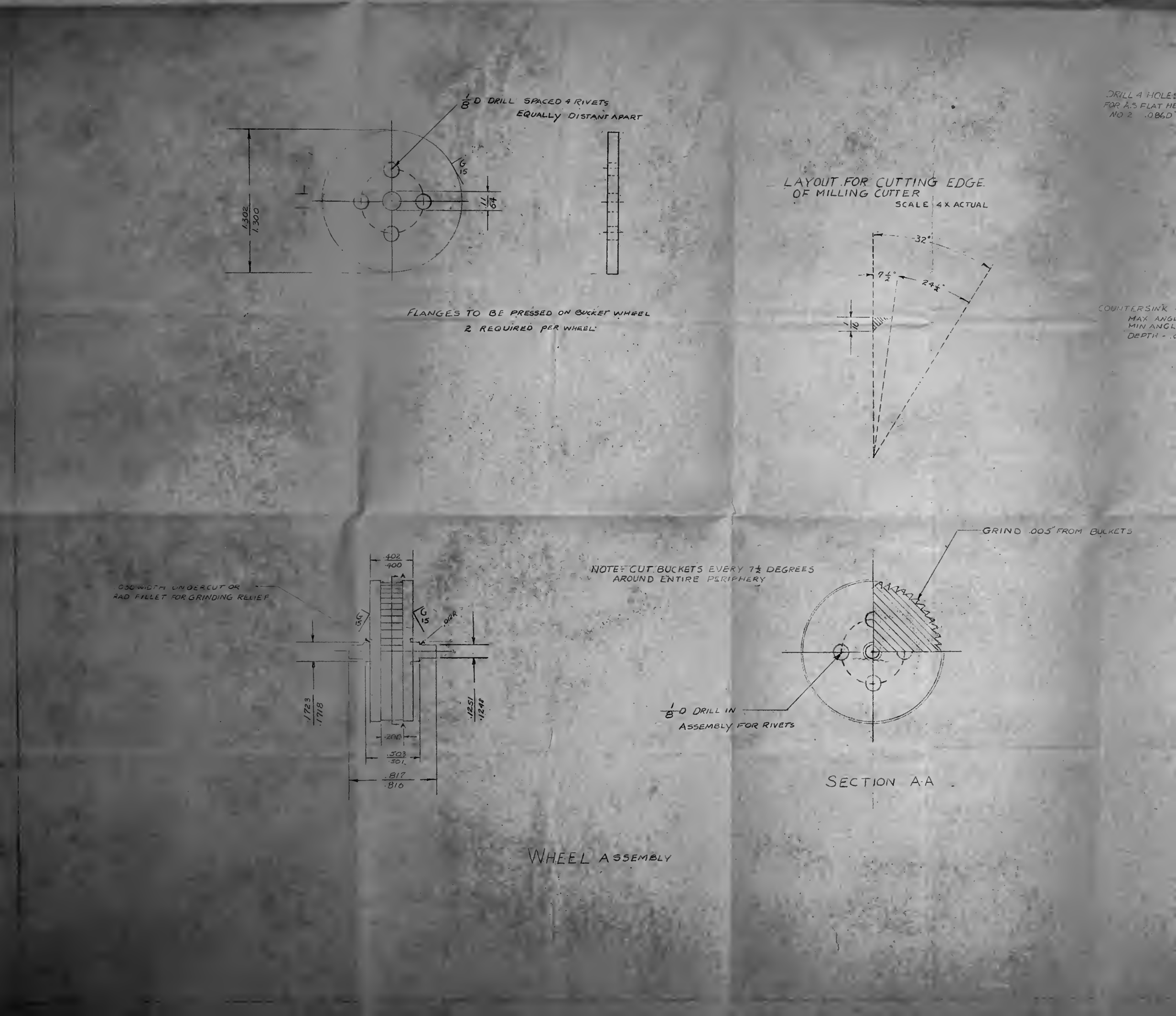
FRONT HALF LEFT WHEEL BRACKET

2 REQUIRED

MATERIAL - DURALUMINUM

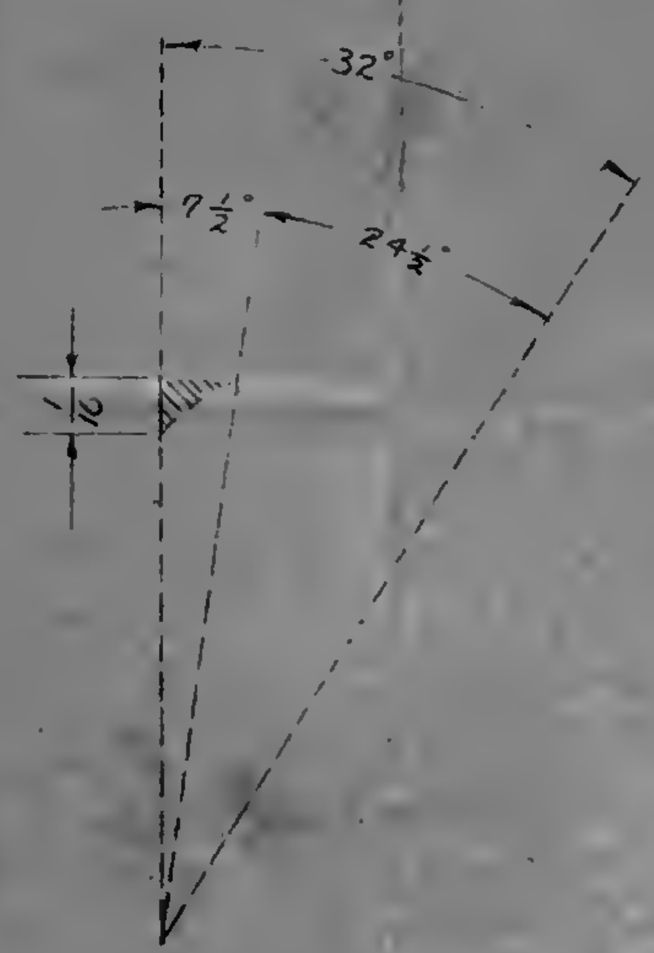
SCALE DOUBLE SIZE

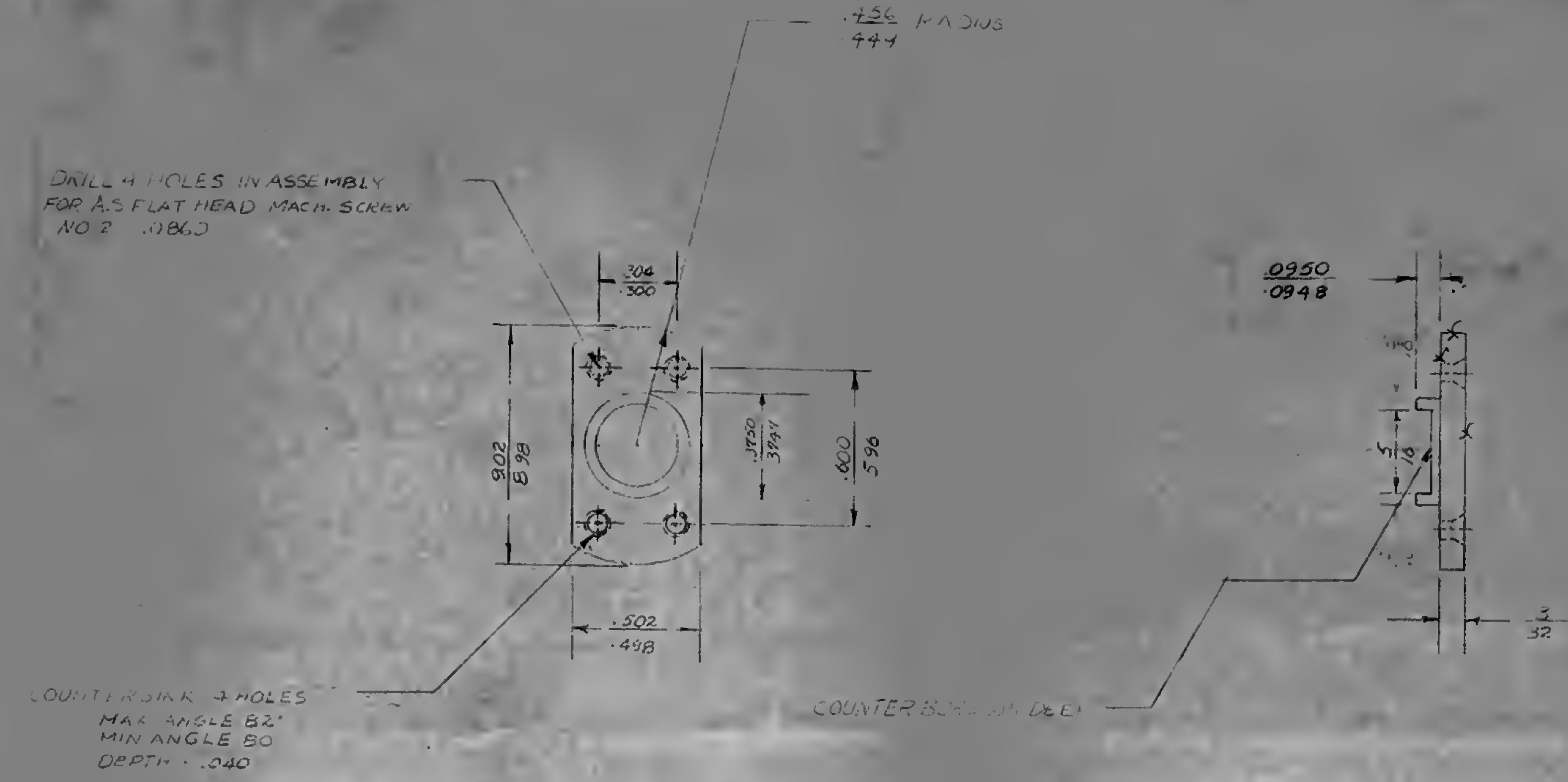
J. E. HAMMER



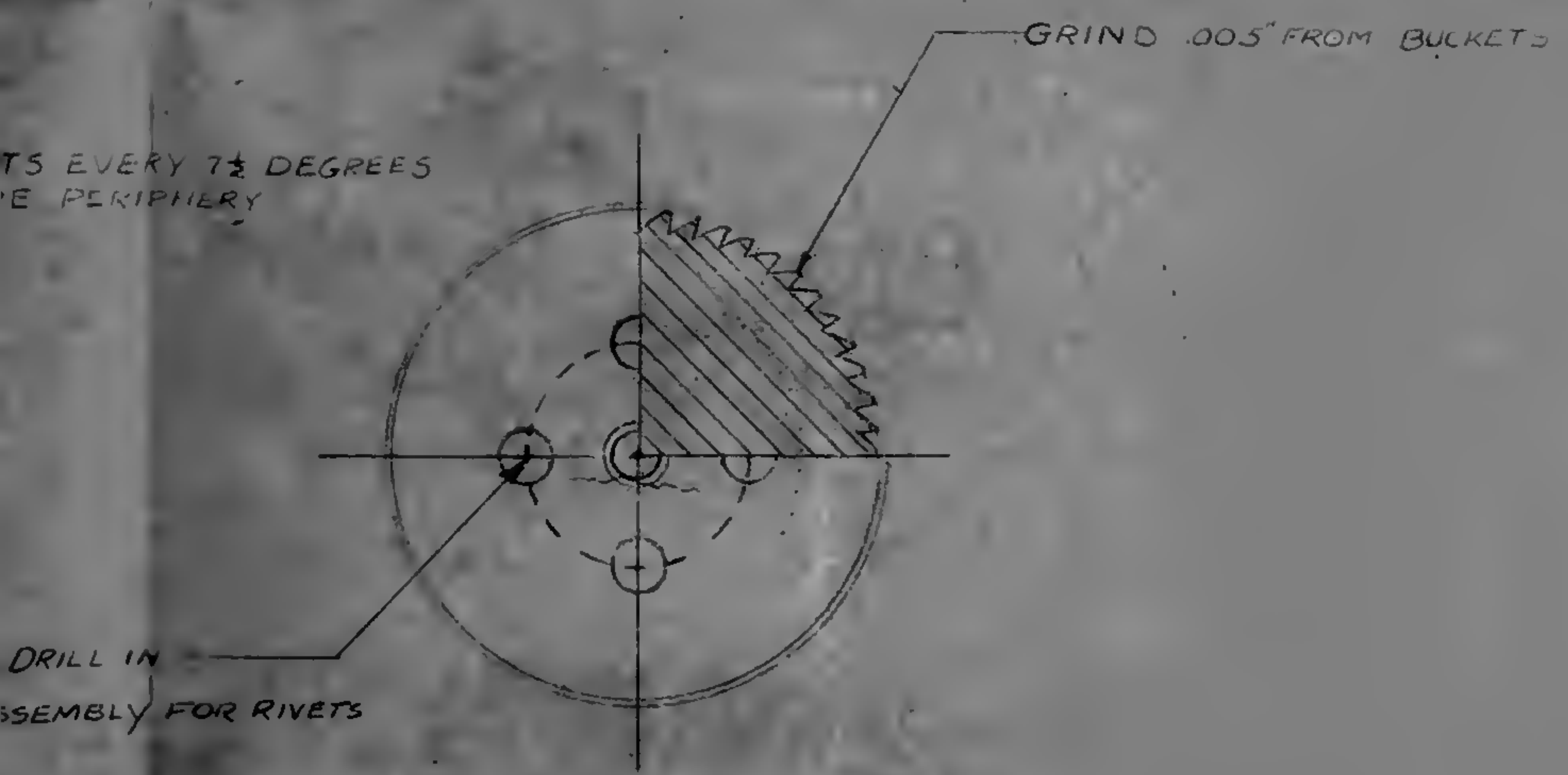
1/8 D DRILL SPACED 4 RIVETS EQUALLY DISTANT APART
1.302
1.300
G 15
11/64
FLANGES TO BE PRESSED ON BUCKET WHEEL
2 REQUIRED PER WHEEL
LAYOUT FOR CUTTING EDGE OF MILLING CUTTER
SCALE 4 X ACTUAL
32°
7 1/2°
24 1/2°
1/16
DRILL 4 HOLES
FOR A.S FLAT HE
NO 2 .086D
COUNTERSINK
MAX ANGL
MIN ANGL
DEPTH - .0
GRIND .005" FROM BUCKETS
NOTE: CUT BUCKETS EVERY 7 1/2 DEGREES
AROUND ENTIRE PERIPHERY
.402
.400
A
G 15
G 15
.02R
.1723
.1718
.1251
.1248
.200
.503
.501
.817
.816
1/8 D DRILL IN
ASSEMBLY FOR RIVETS
SECTION A-A
WHEEL ASSEMBLY

LAYOUT FOR CUTTING EDGE
OF MILLING CUTTER
SCALE 4 X ACTUAL





BEARING COVER



SECTION A-A

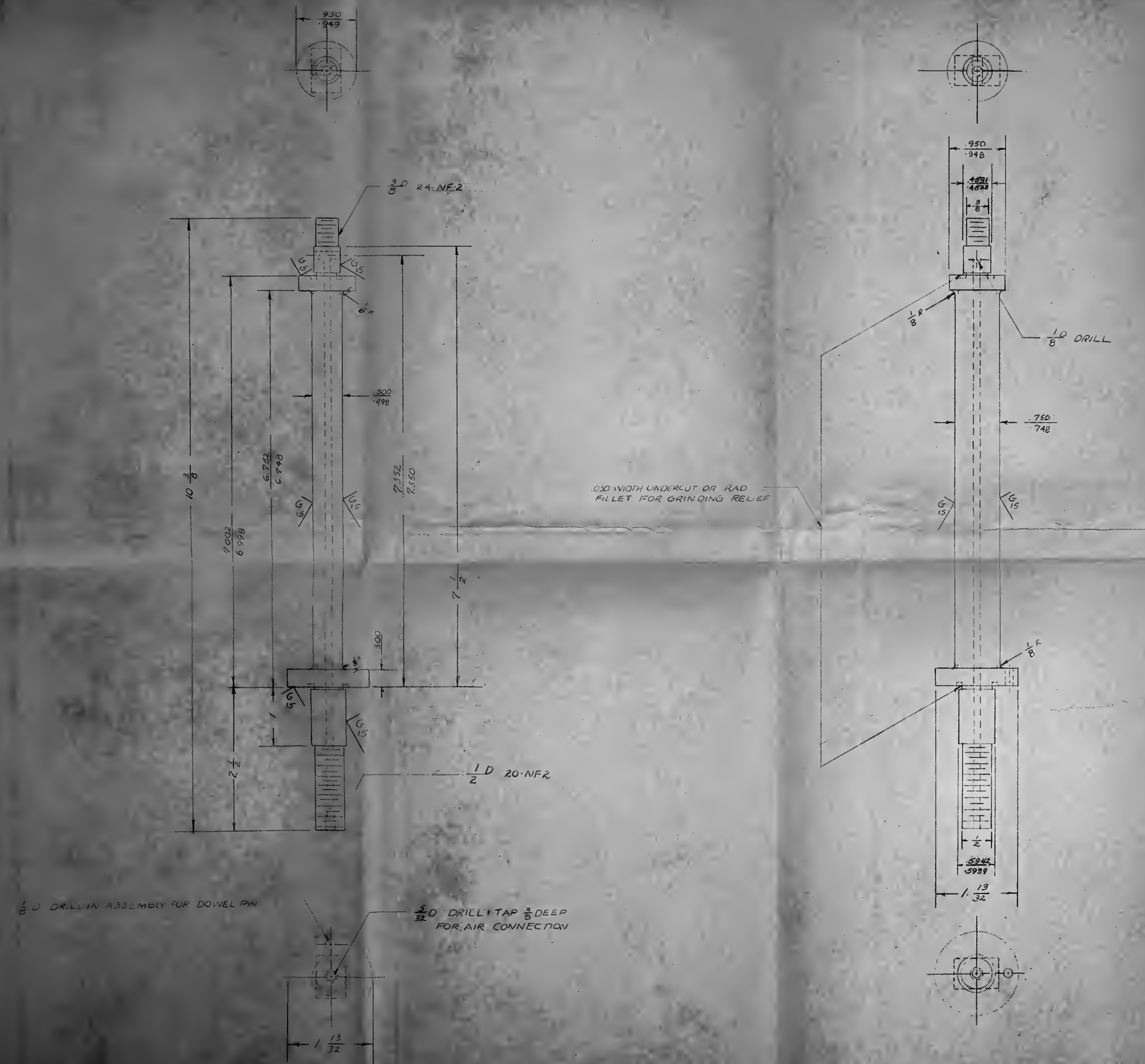
WHEEL
4 PCS REQUIRED
MATERIAL DURALUMINUM

BEARING COVER
8 PCS REQUIRED
MATERIAL DURALUMINUM

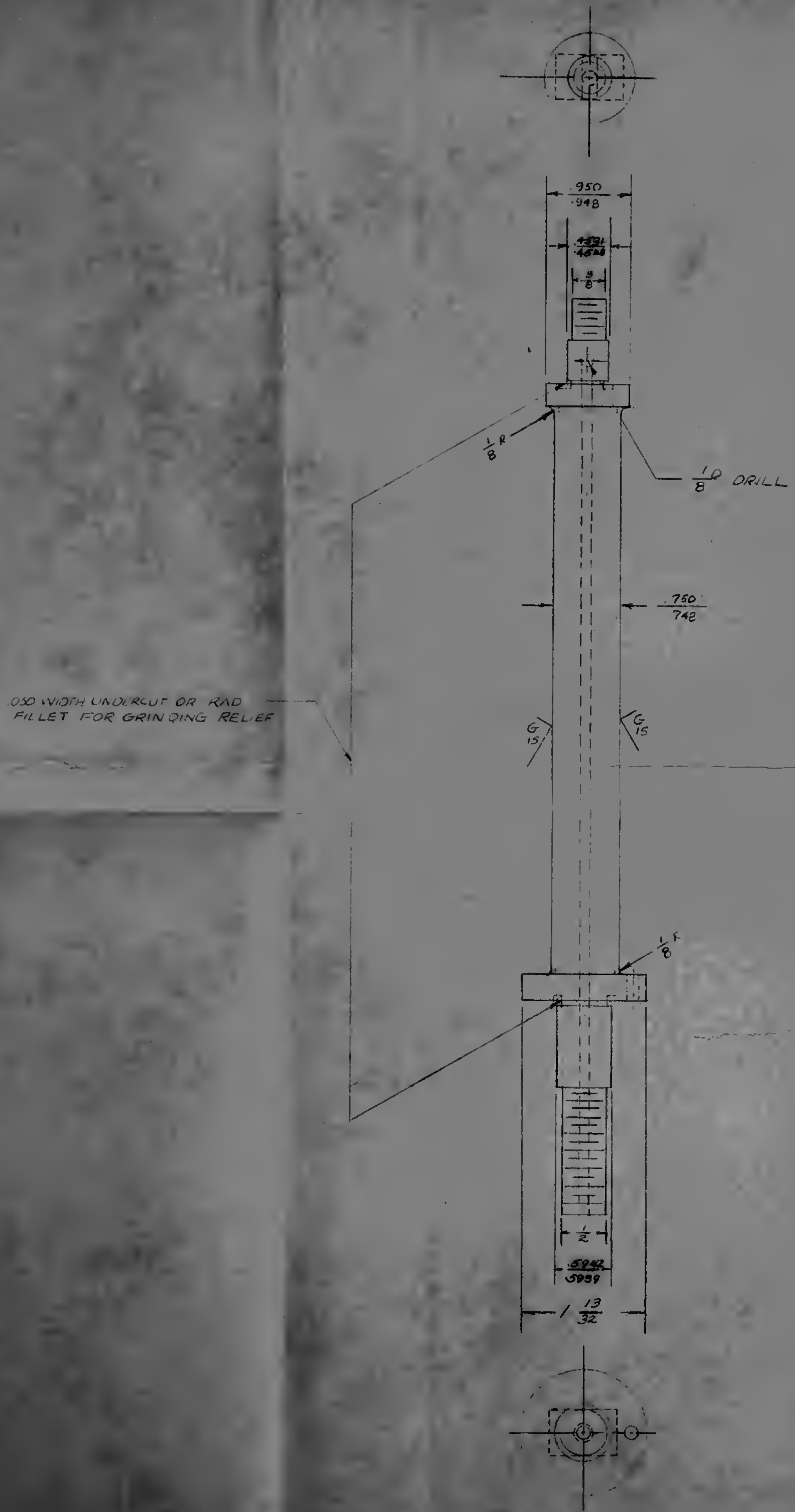
SCALE DOUBLE SIZE

J. E. HAMMERSTONE

U.S.N. POSTGRADUATE SCHOOL



.950
.949
3/8 D 24-NF-2
1/8 R
.500
.498
10 3/8
6.752
6.748
7.002
6.998
7.352
7.350
7 1/2
.300
1
2 1/2
1/2 D 20-NF2
G 15
1/8 D DRILL IN ASSEMBLY FOR DOWEL PIN
5/32 D DRILL & TAP 3/8 DEEP FOR AIR CONNECTION
1 13/32
.950
.948
.4531
.4528
3/8
1/8 D DRILL
.750
.748
.050 WIDTH UNDERCUT OR RAD FILLET FOR GRINDING RELIEF
1/2
.5942
.5939



SUPPORT BEAM
MATERIAL DURALUMIN
2 PCS REQUIRED

SCALE FULL SIZE

J HAMMERSTONE